Srinivasa Ramanujan

a Mathematical Genius

K. SRINIVASA RAO

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Srinivasa Ramanujan

A Mathematical Genius

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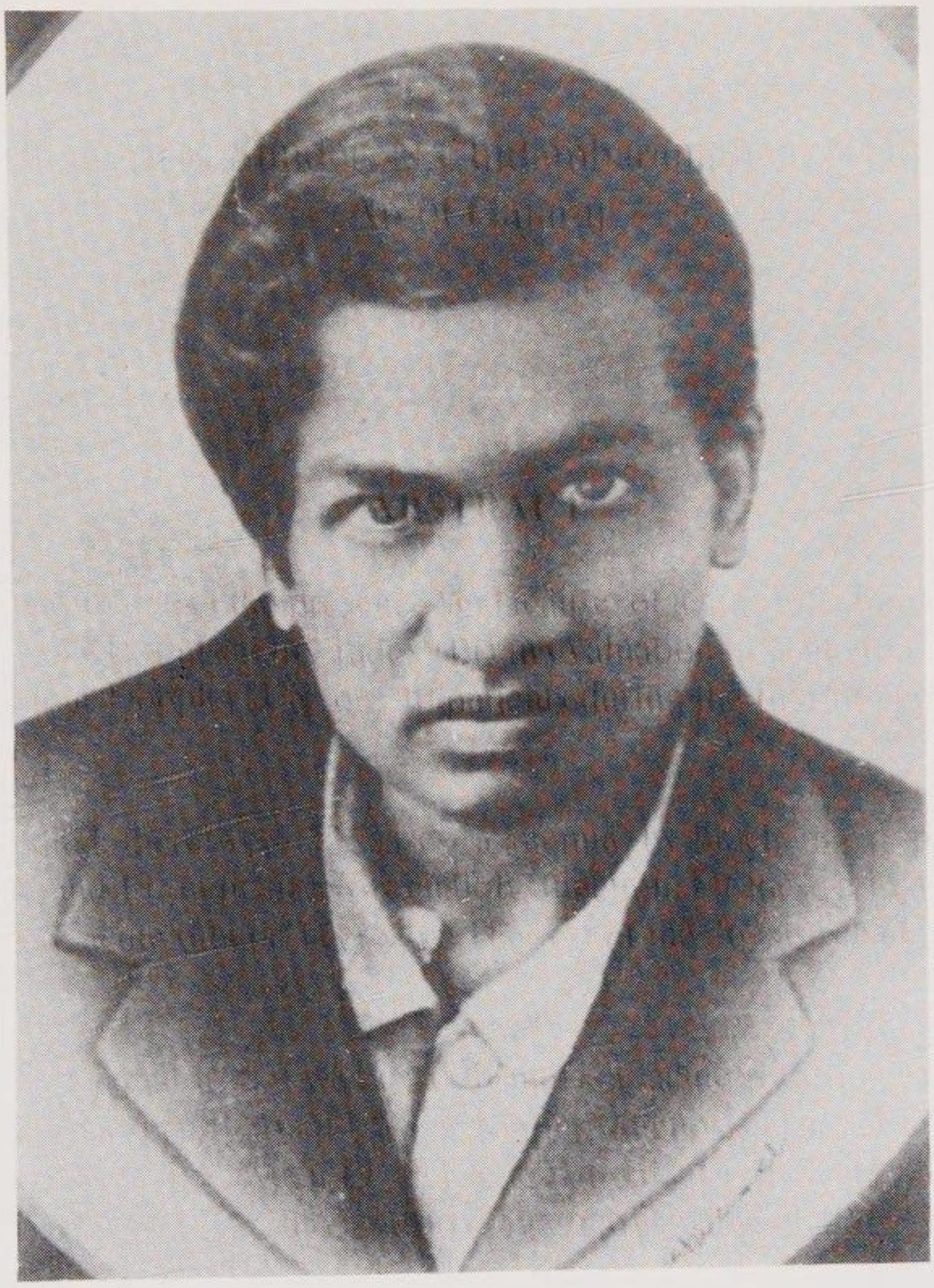
Dedicated to

my mother and sisters

for their unbounded affection



Srinivasa Ramanujan
(Dec. 22, 1887 – April 26, 1920)

Foreword

Several biographies have been written about Srinivasa Ramanujan, generally regarded as India's greatest mathematician. Undoubtedly, the most complete one is *The Man Who Knew Infinity* by R. Kanigel. None of these biographies, however, relate much of Ramanujan's mathematics. In contrast to these other biographies, the present author describes some of Ramanujan's contributions to partitions, hypergeometric series, continued fractions, theta functions, mock theta functions, the Ramanujan τ–function, and other topics.

This biography possesses some further notable features, a few of which we now delineate. First, there is a lengthy discussion about the famed astrophysicist S. Chandrasekhar and the influence of Ramanujan on his life. In particular, the author reproduces an interesting address given by Chandrasekhar at a meeting commemorating the centenary of Ramanujan's birth held at the University of Illinois in June, 1987. Second, the author gives an account of the various busts of Ramanujan. Third, the author provides complete lists of all the papers pertaining to Ramanujan at the Wren Library, Trinity College, Cambridge, and the letters and other material about Ramanujan at the National Archives (New Delhi) and the Tamil Nadu Archives (Chennai).

This book thus significantly adds to our knowledge about Ramanujan and his work.

Urbana-Champaign
January 1998

Bruce C. Berndt
University of Illinois

Preface

This work on Srinivasa Ramanujan who for natural genius has been compared to the all-time greats like Euler, Gauss and Jacobi, is the outcome of my two-part, two-hour lecture on **Srinivasa Ramanujan: his Life and Work**, in several Institutes and Universities in India and abroad (Belgium, England, Germany, Sri Lanka, The Netherlands and U.S.A.).

My first detailed acquaintance with the 'romantic' story of Ramanujan's life and glimpses into his work was through the two volume compilation: **Ramanujan: Letters and Reminiscences** and **Ramanujan: An Inspiration**, memorial numbers, edited by the dedicated Mr. P.K. Srinivasan, published in 1968 to mark the 75th Birth Anniversary of Ramanujan. A better understanding of the genius of Ramanujan came to me with my acquaintance with the group on Special Functions led by Prof. R.P. Agarwal (Lucknow University) when I was directed by Prof. E.C.G. Sudarshan, the then Director of Matscience, in March 1986, to participate in a National Seminar at Gorakhpur University. The subsequent year's Ramanujan centenary celebrations brought me into contact with Professors Richard Askey, George Andrews and Bruce C. Berndt, ardent followers of the work of Ramanujan. Professor Sudarshan encouraged me to collect and collate articles from Professors George E. Andrews, Richard Askey, Bruce C. Berndt, R.P. Agarwal, Arun Verma, S. Bhargava, M.V. Subbarao and also write an article: *Srinivasa Ramanujan: his Life and Work*. These articles were originally intended to appear in a special issue of a popular mathematics journal which had a print run of 5000 copies and subscribed to mostly by colleges in India. It was slated for release on Dec.22, 1987. However, due to circumstances beyond my control, it was released eventually, in Madurai, by Prof. Sudarshan, in March 1988, as a publication of the Madurai-Kamaraj University: **Srinivasa Ramanujan (1887–1920): A Tribute**.

My area of research shifted to Quantum Theory of Angular Momentum and generalized hypergeometric series in the mid-80s and this enabled me to make a few contributions to Number Theory, in the area of Multiplicative Diophantine Equations, and Special Functions (generalized ordinary and basic hypergeometric series, summation theorems for multiple hypergeometric

series, group theoretical basis for transformations of hypergeometric series, etc.). This stimulated my interest in the work of Ramanujan, which was being propagated through the works of Prof. Andrews on the 'lost' Notebook of Ramanujan and the series of books by Prof. Berndt on the original Ramanujan Notebooks.

The inspired biography entitled, **The Man Who Knew Infinity: A Life of the Genius Ramanujan**, by Robert Kanigel, a copy of which was graciously sent to me by Prof. Berndt; visits to Smt. Janaki Ammal Ramanujan, accompanying visiting mathematicians to Madras; several discussions with my journalist friend, late Mr. A. Ranganathan, over the years and the gentle persuasions of Mr. C.A. Reddi, culminated in my visiting the National Archives at New Delhi and the Wren Library of Trinity College, Cambridge (in March and Oct. 95, respectively) to see for myself the original papers of Ramanujan. All this contributed to the general feeling that India 'had done nothing for him'[1] or did anything to preserve for posterity Ramanujan's original papers, notebooks, and other artefacts/memorabilia.

This work is an attempt to provide a brief introduction to the life and work of Srinivasa Ramanujan. It is written with the hope that it will provide information, to the mathematically inclined as well as the mathematics student, on what Ramanujan did to become renowned as a *Natural* Mathematical *Genius*. Dr. Samuel Johnson said:

> *Knowledge is of two kinds. We know a subject ourselves,*
> *or, we know where we can find information about it.*

In that spirit, the author hopes that the reader will get a feeling, after reading this book, for the life and work of Ramanujan and also finds pointers to the sources of his work, for further reading.

The author takes this opportunity to stress the need for a National Science

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[1] J.M. Whittaker's statement, in Feb. 1979, that the Trinity College, Cambridge, was the right place for the Ramanujan material, rather than India which had done nothing for him (quoted in Kanigel [X]). This has been refuted by Berndt and Rankin [XI] who assert that ... *it is clear that his Indian colleagues had done all that was within their powers to assist him.*

Museum in Madras which will portray to the comman man, the interested student of Science and the specialist as well, the evolution and growth of Physical Laws and their Mathematical Beauty; depict the work of our country's great scientists like Ramanujan, Raman, Satyendranath Bose, Chandrasekar, ...; collect and preserve the artefacts and memorabilia of great scientists and above all be a source of inspiration to instil a sense of pride and kindle the urge to a better understanding of Nature through Science.

Madras - 600 113
March 1998

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The Institute of Mathematical Sciences

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The author acknowledges gratefully the following for permission to reprint the copyrighted material: The Master and Fellows of Trinity College, Cambridge for the S. Ramanujan card catalogue in the Wren Library; Cambridge University Press for quotations from *Collected Papers of Srinivasa Ramanujan* (1927), *Ramanujan: Twelve Lectures on subjects suggested by his life and work* by G.H. Hardy (1940) and *A Mathematician's Apology* by G.H. Hardy (1967); Dr. S. Ramaseshan and the Indian Academy of Sciences for the quotation from Pathrika and the Academic Press for the remarks of Dr. S. Chandrasekhar *On Ramanujan.* The author's failure to get the required permission for use of any other copyrighted material included in this work is inadvertent and will be set right in future editions, on receipt of intimation to the publisher of such omissions.

The author acknowledges with thanks the encouragement and support received from the successive Directors of MATSCIENCE, the Institute of Matheamatical Sciences, Madras – Prof. Alladi Ramakrishnan (Founder), Prof. E.C.G. Sudarshan and Prof. R. Ramachandran – and my colleagues and friends at this Institute (now or earlier), in particular, (late) Prof. R. Vasudevan; Professors G. Rajasekaran, N.R. Ranganathan, V. Radhakrishnan, R. Parthasarathy (Matscience); K. Ananthanarayanan (San Diego), T. S. Santhanam (St. Louis), A. Sundaram (Madurai), N. R. Nandakumar (Delaware), M. R. Subrahmanya (Tiruchirapalli) and (late) G. N. Keshavamurthy (Shimoga). I am grateful to Prof. Bruce C. Berndt for sending me copies of the books: *Ramanujan Revisited* (edited); *The Man Who Knew Infinity: a Life of the Genius Ramanujan* by Robert Kanigel; his *Ramanujan Notebooks, Part III* and *Ramanujan: Letters and Commentary*, edited by him and Robert A. Rankin, and for innumerable e-mail correspondences on various aspects concerning Ramanujan over the past few years. I am also grateful to Profs. R. Askey, G.E. Andrews, (late) S. Chandrasekhar ('Chandra') and V.S. Ramamurthy, who inspired and encouraged me and to Prof. R.H. Dalitz, who visited Madras, in March 1996, for his enthusiasm to share information on Chandra and Ramanujan.

To (late) Mr. A. Ranganathan, my journalist friend, I am indebted for several exchanges of ideas, suggestions and conversations, mostly over the telephone, which motivated me to acquire information from several sources and even attempt to get copies of two letters (from Dr. S. Chandrasekhar to the then Prime Minister Mr. Jawaharlal Nehru and the latter's reply) in the Nehru Memorial Museum and Library. Mr. C. A. Reddi's efforts in getting the papers on Ramanujan from the Tamil Nadu Archives, and his persuasion, made me visit the National Archives at New Delhi and the Wren Library of Trinity College, Cambridge. I am thankful to Prof. Rankin for his spontaneity in writing to the Librarian, Dr. David McKitterick, and getting me a Reader's Table at the Wren Library, for two days in October 1995.

This camera ready copy produced in LaTeX had its origins in the summer of 1995 and the first draft was prepared in TeX by the trainees of Scientific Publishing Services (P) Ltd., during a course of lectures on TeX, along with two of my colleagues, Profs. R. Jagannthan and R. Parthasarathy. I am thankful to Profs. R. Askey, B.C. Berndt, R. Jagannathan, V. Lakshminarayanan, R. Sridhar and J. Van der Jeugt, for their comments and suggestions after reading preliminary versions of the book; to Ms. Radhika Vathsan and Mr. S. Srinivasa Rao for help in scanning the photos and to Mr. I. Suresh for his help in the preperation of the Notes section. I acknowledge the spontaneity and help of the Librarian of Matscience, Mr. K.S. Santhanagopalan and Mr. G. Venkatesan (Deputy Librarian) and the supporting staff, in my literature searches and Dr. G. Subramoni for help with the computer system of the Institute. The general administrative support of the Institute's team under the leadership of Mr. G. Sethuraman, Chief Administrative Officer, is acknowledged with thanks.

Last but not the least, I owe a lot to my family – my mother Mrs. Lakshmikanthamma Vallabheswar Rao, my brother Prof. K. Sivarama Sastry (who corrected the proofs of the first TeX draft of this work), my sisters (late) Ms. K. Bhuvaneswari and Ms. K. Vijayalakshmi Sundari and my wife, Dr. Geetha, and sons K.S. Aravind and K.S. Anand – for providing me all the essential support and encouragement in my pursuit of information and knowledge.

This work has been partly supported by the European Commission (contract No. CI1*-CT92-0101) and I wish to thank Profs. G. Vanden Berghe, J. Van der Jeugt and H. De Meyer of the University of Ghent, Belgium, for the excellent hospitality and help of every kind to make my visits to Ghent, during the 4-year project period, memorable. Considerable amount of work on this manuscript was done during the author's stay at Ghent (Sept.–Nov. 1997).

Madras - 600 113
March 1998

K. Srinivasa Rao
The Institute of Mathematical Sciences

Contents

Foreword i

Preface iii

Acknowledgements vii

1 Life of Ramanujan 1

2 Ramanujan's Mathematics: Glimpses 39

3 Ramanujan's Notebooks 74

4 Hardy on Ramanujan 92

5 Chandra and Ramanujan 118

6 Books and Busts 137

7 What is where 160

Appendix 1. Research publications of Ramanujan 182

Appendix 2. Wren Library Card Catalogue and Papers of Ramanujan 186

Appendix 3. File on S.Ramanujan at the National Archives and at the Tamil Nadu Archives 203

References* 209

Notes 219

*Principal references are numbered in Roman numerals throughout this work

1. Life of Srinivasa Ramanujan

Srinivasa Ramanujan, the brilliant twentieth century Indian mathematician, has been compared with all-time greats like Euler, Gauss and Jacobi, for his **natural** mathematical **genius**. It may be impossible to define who a *mathematical genius* is, or, genius for that matter. But that does not prevent us from recognizing the work of a genius, the rarest of the rare – like the Himalayan peaks or the Niagara falls – which stands out in any field of human activity. Despite his short life span, Ramanujan left behind an incredibly vast and formidable amount of original work, which has greatly influenced the development and growth of some of the best research work in mathematics of this century. In this chapter we give a brief account of his life.

Early years

Ramanujan was born on December 22, 1887, at Erode, to Komalathammal and K. Srinivasa Iyengar, who was an accountant to a cloth merchant in Kumbakonam on a modest salary. He was the first of three sons. He was sent to Kangeyam Primary School in Kumbakonam at the age of seven. He stood first in the Tanjore District Primary Examinations held in November 1897, and this entitled him to a half-fee concession in the Town High School at Kumbakonam, where he studied from 1898 to 1903. He passed the Matriculation Examination of the University of Madras (1904).

During his school days, he impressed his classmates, senior students and teachers with his extraordinary intuition and astounding proficiency in several branches of mathematics — viz. arithmetic, algebra, geometry, number theory and trigonometry. In later years a friend of his recounted the following incident [1]: In an arithmetic class on division, the teacher said that if three bananas were given to three boys, each boy would get a banana. The teacher generalized this idea. Ramanujan is said to have asked:

> *Sir, if no banana is distributed to no student,*
> *will everyone still get a banana?*

Another friend who took private tuition from Ramanujan also recalled [2]

that Ramanujan used to ask about the value of zero divided by zero and then answer that it can be anything since the zero of the denominator may be several times the zero of the numerator and vice versa and that the value cannot be determined. Once a senior school student [1], posed the problem:

"if $\sqrt{x} + y = 7$ and $x + \sqrt{y} = 11$, what are x and y?",

to Ramanujan who was in the fourth year at school. Ramanujan's immediate reply[1] — to this question which was expected to be tackled by only a sixth year student — that $x = 9$ and $y = 4$ won for him a friend who in later years took him to the collector of Nellore. The senior mathematics teacher of the school (Ganapathy Subbier) had such confidence in Ramanujan's ability that year after year he entrusted Ramanujan with the task of preparing a conflict free time-table [3] for the school. Ramanujan won prizes for his outstanding performances in mathematics and mastered Loney's Trigonometry, Part II, in his fourth year at school. He won many prizes [4] in his second, fourth and sixth years at High School.

To augment the family income, Ramanujan's mother took in a couple of students from Tirunelveli and Tiruchirapalli as boarders. Noticing Ramanujan's precocity in mathematics these undergraduate students are purported to have given him an elementary introduction to all branches of mathematics. In 1903, through these friends from the Kumbakonam Government College, Ramanujan obtained G.S. Carr's: *A Synopsis of Elementary Results, a book on Pure Mathematics*, which contained propositions, formulae and methods of analysis with abridged demonstrations published in 1886. Prof. G.H. Hardy [IV] says about George Shoobridge Carr, formerly Scholar of Gonville and Caius College, Cambridge:

> *Carr himself was a private coach in London, who came to Cambridge as an undergraduate when he was nearly forty, and was 12th Senior Optime in the Mathematics Tripos of 1880 (the same year in which he published the first volume of his book). He is now completely forgotten, even in his own College, except in so*

---

[1]Setting $x = m^2$, $y = n^2$, taking the difference between the two simultaneous equations and factorizing: $(m-n)(m+n-1) = 4$, which has integer solutions only for $m = 3, n = 2$ or $x = 9, y = 4$.

> *far as Ramanujan has kept his name alive. ...* Carr's book *covers roughly the subjects of Schedule A of the present Tripos*[2] *(as these subjects were understood in Cambridge in 1880), and is effectively a "synopsis" it professes to be. It contains the enunciations of 6165 theorems, systematically and quite scientifically arranged, with proofs which are often little more than cross-references and are decidedly the least interesting part of the book. All this is exaggerated in Ramanujan's famous notebooks*[3] *(which contain no proofs at all), and any student of the notebooks can see that Ramanujan's ideal of presentation has been copied from Carr's.*

Prof. Richard Askey has pointed out[4] the fact that in Carr's 'Synopsis', while formulae in each chapter are numbered continuously, the numbering for each chapter starts with 100, 200, etc., discontinuously. Numbers are skipped in intermediate places, as well as in certain chapters, and it is conjectured that this was done perhaps with a view to the possibility of adding new material in approximately the right places in future editions of the book. In all about 1300 numbers are skipped and so, though the last entry bears the number 6165, referred to by Hardy, the total number of formulae is only about 4865. These formulae, presented by Carr without proofs, are in algebra, trigonometry, analytical geometry and calculus. This book is similar to the later day compilations like that of, say, *Table of Integrals, Series, and Products, by I.S. Gradshteyn and I.M. Ryzhik* (5th edition, Academic Press, New York, 1994). Prof. P.V. Seshu Aiyar and Mr. R. Ramachandra Rao, in their biographies of Ramanujan (see [III]) state that:

> *It was this book which awakened his genius. He set himself to establish the formulae given therein. As he was without the aid of other books, each solution was a piece of research so far as he was concerned.*

It is the considered opinion of many (cf. Kanigel [X], p.57) that in proving one formula, he discovered many others and thus, Ramanujan laid for himself a foundation for higher mathematics. Also, during the years 1904 to 1907, he started noting the formulae down in his Notebooks.

---

[2] An excellent exposition of the Mathematical Tripos of Cambridge, a coveted examination which Hardy took but in later years denounced, can be found in Kanigel [X].

[3] Chapter 3 of this work is devoted to these Notebooks.

[4] Private communication.

Formal Education

The first public recognition of his extraordinary prowess came when he was awarded a special prize — the Sri K. Ranganatha Rao Prize — at the annual prize distribution ceremony of the Town High School, in 1904, for proficiency in mathematics. Ramanujan passed his Matriculation Examination in 1904 and joined the Government Arts College in Kumbakonam. As a result of his success in a competitive examination in Mathematics and English composition, he secured the Junior Subrahmanyam Scholarship. In the F.A. (First Examination in Arts) Class, Ramanujan had to study English, Sanskrit, Mathematics, Physiology and the History of Rome and Greece. Partly due to his pre-occupation with researches into mathematics, he neglected the study of other subjects. He went to his mathematics lecturer with a number of original and very ingenious results in finite and infinite series. Prof. P.V. Seshu Aiyar exhorted him but advised him not to neglect the study of other subjects. Unfortunately, he did not pass in English and hence was not promoted to the senior F.A. class in January 1905. He lost his scholarship. His mother, who played a domineering role in his life, tried to persuade the Principal of the Government Arts College to take note of Ramanujan's extraordinary mathematical ability and appealed for a continuance of the scholarship, but to no avail.

Ramanujan's failure to get promoted to the senior F.A. class marked the beginning of a very trying period in his life. In 1906, he set out from Kumbakonam, for the first time, and joined Pachaiyappa's College in Madras, in the F.A. class again. After about three months, he fell ill and discontinued his studies. However, he appeared privately for the F.A. examination in 1907. Though he secured a centum in mathematics, he failed to secure pass marks in other subjects. This marked the end of his formal education.

Years of adversity

> *'The years between 18 and 25 are the critical years in a mathematician's career.... During his five unfortunate years (1907-1912) his genius was misdirected, sidetracked and to a certain extent distorted'.* (Hardy [IV]).

Fig. caption: Mr. R. Ramachandra Rao, Mr. V. Ramaswami Aiyar and Prof. P.V. Seshu Aiyar – admirers of Ramanujan – were Presidents of the IMS. Prof. E.W. Middlemast, Mathematics Professor at the Presidency College, Madras, recommended Ramanujan, in 1911, for a position at the Madras Port Trust as *a young man of quite exceptional capacity in Mathematics ([XI], p.7).* He was also a President of the IMS for a year.

Despite the pecuniary circumstances and the stresses and strains of day-to-day existence, Ramanujan started noting down his own results in Notebooks. A college mate [5] of Ramanujan has stated that during his collegiate years, Ramanujan taught him the method of constructing Magic Squares[5], the subject of the first chapter of his first and second Notebooks. The interest in this subject dates from his school days and is disconnected from the subject matter of the remainder of the Notebooks. Ramanujan's investigations in continued fractions and divergent series started during this period. His betrothal to nine year old Janaki was in 1908 and his wedding took place near Karur, in 1909. Robert Kanigel [X], in his biography of Ramanujan, constructs a vivid account of this marriage arranged by his mother Komalathammal, not approved by his father, and dramatizes the foreboding of the impending disaster through the omens preceding the wedding, which was on the brink of being called off due to the late arrival of the bridegroom's party.

During 1906 – 1912, Ramanujan was constantly in search of a benefactor and a job to earn enough for his needs. He tutored a few students in mathematics, in Kumbakonam and later even sought employment as a tutor in mathematics. Disappointed at the lack of recognition, during this trying period, Ramanujan had bemoaned to a friend [2] that he was probably destined to die in poverty like Galileo! This was not to be.

In 1910, Ramanujan sought the patronage of Prof. Ramaswamy Iyer – the founder of Indian Mathematical Society – who was at Salem and asked for a clerical job in his office. The only recommendation Ramanujan had was his Notebooks which by then contained several results on Magic Squares, prime numbers, infinite series, divergent series, Bernoulli numbers, Riemann zeta function, hypergeometric series, partitions, continued fractions, elliptic functions, modular equations, etc. A scrutiny of the entries in the Notebooks was sufficient to convince Prof. Ramaswamy Iyer [6] that Ramanujan was a gifted mathematician and he

> *had no mind to smother his (Ramanujan's) genius by an appointment in the lowest rungs of the revenue department.*

So, he sent Ramanujan back to Madras with a letter of introduction to

---

[5]Probably Ramanujan's expertise in preparing the conflict free time tables for his School inspired him to a study of these Magic Squares.

Prof. P.V. Seshu Aiyar, then at the Presidency College, Madras. Prof. Seshu Aiyar, who had known Ramanujan as a student at the Government Arts College, Kumbakonam, when he himself was employed there as a lecturer of mathematics, was meeting him after a gap of four years and was greatly impressed with the contents of the *well-sized Notebooks.* So he gave Ramanujan

> *a note of recommendation to that true lover of mathematics, Dewan Bahadur R. Ramachandra Rao, who was then the District Collector at Nellore.*

The turning point

With the help of a friend (R. Krishna Rao) [1] and a nephew of Dewan Bahadur Ramachandra Rao, Ramanujan went to Nellore in December 1910. This was the turning point in Ramanujan's life. Ramachandra Rao states [7] that *in the plentitude of my mathematical wisdom, I condescended to permit Ramanujan to walk into my presence.* At that time, Ramanujan appeared to Ramachandra Rao as

> *a short uncouth figure, stout, unshaved, not over-clean, with one conspicuous feature — shining eyes — walked in, with a frayed Notebook under his arm.... He was miserably poor. He had run away from Kumbakonam to get leisure in Madras to pursue his studies. He never craved for any distinction. He wanted leisure, in other words, simple food to be provided for him without exertion on his part and that he should be allowed to dream on.*

Though Ramachandra Rao gave him a patient hearing, he took a few days to look into the Notebooks of Ramanujan. At their fourth meeting, when Ramanujan confronted Ramachandra Rao with a letter from Prof. Saldhana of Bombay appreciating the genuineness of his work, Ramachandra Rao started to feel that Ramanujan's work must be examined in depth by eminent mathematicians. Ramachandra Rao himself states [7] that Ramanujan led him

> *step-by-step to elliptic integrals and hypergeometric series and at last to his theory of divergent series not yet announced to the world*

and this converted him into a benefactor who undertook to underwrite Ramanujan's expenses at Madras for some time.

Prof. Seshu Aiyar also communicated the earliest contributions of Ramanujan to the Journal of the Indian Mathematical Society (I.M.S.) in the form

of questions. These appeared in 1911 and in his brief and illustrious career Ramanujan proposed in all 59 questions or solutions to questions in this journal. The first fifteen page article entitled: *Some properties of Bernoulli numbers*[6] appeared in the same 1911 volume of the journal of the I.M.S. In it Ramanujan stated eight theorems embodying arithmetical properties of the Bernoulli numbers, indicating proofs for three of them; two theorems are stated as corollaries of two others, while three theorems are stated as mere conjectures. Prof. P.V. Seshu Iyer states ([I], p. 125):

> *Ramanujan's methods were so terse and novel and his presentation was so lacking in clearness and precision, that the ordinary reader, unaccustomed to such intellectual gymnastics, could hardly follow him.*

Mr. M.T. Narayanaiengar, one of the early editors of the Journal of the Indian Mathematical Society states that this first article of Ramanujan:

> *... attracted considerable attention. It is however a sad confession to say that the Editor's work in connection with Ramanujan's contribution was by no means light. Ramanujan saw intuitively many things and could not bring himself to explain and the first article had to be referred back to him no less than three times.*

Ramanujan lived in a small house called *'Summer House'* in Sami Pillai Street, Triplicane, Madras, accepting reluctantly a monthly financial assistance from the collector of Nellore for about a year. Later he declined this help and from Jan. 12 to Feb. 21, 1912, he worked as a clerk in the Accountant General's Office. Not satisfied with this job, Ramanujan applied for and secured a post in the Accounts Section (Class III, Grade IV) in the Madras Port Trust, with the help of Mr. S. Narayana Iyer, the Manager of Port Trust, who was the treasurer of the IMS and a friend of Profs. V. Ramaswamy Aiyar and P.V. Seshu Aiyar.

Mr. Narayana Aiyer was a good mathematician and was a great source of support to Ramanujan. He was not only instrumental in Ramanujan being

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[6]The first few Bernoulli numbers [27] are: $B_0 = -1, B_2 = \frac{1}{6}, \quad B_4 = -\frac{1}{30}, \quad B_6 = \frac{1}{42}, \quad B_8 = \frac{1}{30}, B_{10} = \frac{5}{66}, \quad B_{12} = \frac{691}{2730}, \quad B_{14} = \frac{7}{6}, \cdots B_{38} = \frac{29299939913841559}{6}$.

offered a job in the Madras Port Trust, but also in securing for Ramanujan the life-long support of Sir Francis Spring to Ramanujan's career. Narayana Aiyer's son N. Subbanarayanan relates the role his father played in the career of Ramanujan [I, p. 112]:

> *To illustrate the depth of his genius, I can quote one of the incidents which happened during the stay of Sri Ramanujan with my father when we were living in No. 580, Pycrofts Road, Triplicane. During that period, every night both Ramanujan and my father used to work Mathematics in two big sized slates sitting on a small parapet upstairs. This used to be carried on till about 11.30 p.m. and was a source of nuisance to other inmates of the house who used to sleep in the adjoining rooms. I distinctly remember the noises of the slate pencils which used to be a background music for my sleep. Several nights I have seen Sri Ramanujan get up at 2 o'clock in the night and note down something in the slate in the dull light of a hurricane lamp. When my father asked him what he was writing, he used to say that he worked out mathematics in his dreams and now he was jotting the results in the slate to remember them. Evidently, this goes to show that he must have had a remarkable power of subconscious working and grasp of intellectual strides in mathematics. Peculiarly enough, my father used to ask Ramanujan some doubts in these jottings. Between one step and another step there used to be a big gap. My father, being a fairly good mathematician himself, was unable to capture the strides of Ramanujan's discoveries. He used to tell him, "When I am not able to understand your steps, I do not know how other mathematicians of a critical nature will accept your genius. You must descend to my level and write at least ten steps between the two steps of yours." Sri Ramanujan used to say, "When it is so simple and clear to me, why should I write more steps?" But somehow my father slowly got him round, cajoled him and made him write some more, though it used to be a mighty task of boredom to him.*

Mr. Narayana Iyer requested for and obtained the slate used by Ramanujan, as a memento (in exchange of his slate) and it is perhaps still a heirloom with the family of Mr. Subbanarayanan. It is interesting to note that

Mr. Narayana Iyer used to send his son Subbanarayanan to get a book on Elliptic Functions from the library of the University of Madras [I, p.115].

Dewan Bahadur Ramachandra Rao also wrote to Sir Francis Spring, Chairman of Madras Port Trust, about Ramanujan and Ramanujan's entry into Port Trust, on March 1, 1912, may well be considered as **the** turning point in his career prospects. He held this post for 14 months. His wife joined him during this period and Ramanujan shifted his residence to Saiva Muthiah Mudali Street in George Town. This period marked the beginning of the appreciation of his scholarship and researches in mathematics.

Dewan Bahadur Ramachandra Rao induced Prof. C.L.T. Griffith of the Engineering College, Madras to take interest in Ramanujan and he in turn wrote [8] in November 1912, to Sir Francis Spring, the Chairman of Madras Port Trust about the 'very poor accountant' who was *'a most remarkable mathematician'* and asking him to keep Ramanujan *'happily employed until something can be done to make use of his extraordinary gifts'*. Prof. Griffith also wrote to Prof. M.J.M. Hill, of University College, University of London, on Ramanujan's work and he received a reply in December 1912. Unfortunately, Prof. Hill [9] could not find time to study the results. He observed that *'the book which will be most useful to him is Bromwich's Theory of Infinite Series, published by Cambridge University Press (or Macmillan)'* and gave advice as to how Ramanujan could get his papers published. In a sequel to this reply, dated 7 December 1912, Prof. Hill wrote to Prof. Griffith [XI, p.18]:

> *"Mr. Ramanujan is evidently a man with a taste for Mathematics, and with some ability, but he has got on the wrong lines. He does not understand the precautions which have to be taken in dealing with divergent series, otherwise he could not have obtained the erroneous results you send me, viz.:–*

$$\begin{aligned} 1+2+3+\cdots+\infty &= -\frac{1}{12}, \\ 1^2+2^2+3^2+\cdots+\infty^2 &= 0, \\ 1^3+2^3+3^3+\cdots+\infty^3 &= \frac{1}{240}. \end{aligned}$$

The sums of n terms of these series are:

$$\frac{1}{2}n(n+1), \quad \frac{1}{3}n(n+\frac{1}{2})(n+1), \quad \frac{1}{2}[n(n+1)]^2$$

and they all tend to ∞ as n tends to ∞. I do think you can do no better for him than to get him a copy of the book I recommended, Bromwich's Theory of Infinite Series, published by Macmillan and Co., who have branches in Calcutta and Bombay. Price 15/- net."

Ramanujan published two short notes, one *On question 330 of Professor Sanjana* and another a *Note on a set of simultaneous equations*, in the IMS journal, in 1912. When Ramanujan approached Prof. Seshu Aiyar with some theorems on Prime Numbers, his attention was drawn to G.H. Hardy's Tract on *Orders of infinity.* Ramanujan observed that ([III], p.xxii);

on p.36 of this Tract "the exact order of $\rho(x)$ [defined by the equation:

$$\rho(x) = \pi(x) - \int_2^x \frac{dt}{log\ t}$$

where $\pi(x)$ denotes the number of primes less than x,] has not yet been determined".

In other words, that "no definite expression has yet been found for the number of prime numbers less than any given number". Ramanujan told Prof. Seshu Aiyar that he had discovered a result which gave the order of $\rho(x)$. This made Prof. Seshu Aiyar suggest communication of this and other results to Mr. G.H. Hardy – a Fellow of the Royal Society and Cayley Lecturer in Mathematics at Cambridge – a world famous mathematician, who was ten years Ramanujan's senior.

The first (11-pages) letter of Ramanujan to Hardy, dated Jan. 16, 1913, is a historic letter. While more about the contents of this letter and its impact on Hardy will be presented in chapter 2 of this work, in this section, in connection with the comments of Prof. Hill on divergent series, we draw attention here only to what Ramanujan ([XI], p.290) stated on divergent series, in that letter to Hardy:

I have got theorems on divergent series, theorems to calculate the

convergent values corresponding to the divergent series, viz.:

$$\begin{aligned} 1-2+3-4+\cdots &= \frac{1}{4}, \\ 1-1!+2!-3!+\cdots &= .596\ldots, \\ 1+2+3+4+\cdots &= -\frac{1}{12}, \\ 1^3+2^3+3^3+4^3+\cdots &= \frac{1}{240}. \end{aligned}$$

Theorems to calculate such values for any given series (say $1-1^1+2^2-3^3+4^4-5^5+\cdots$*), and the meaning of such values.*

I have also dealt with such questions 'When to use, where to use, and how to use such values, where do they fail and where do they not?'

Hardy's prompt reply dated Feb.8, 1913, was the starting point of Ramanujan's recognition by the Western world of Mathematicians and this most encouraging letter read [XI, p.46]:

Dear Sir,

I was exceedingly interested by your letter and by the theorems which you state. You will however understand that, before I can judge properly of the value of what you have done, it is essential that I should see proofs of some of your assertions.

Your results seem to me to fall into roughly 3 classes:

(1) there are a number of results which are already known, or are easily deducible from known theorems;

(2) there are results which, so far as I know, are new and interesting, but interesting rather from their curiosity and apparent difficulty than their importance;

(3) there are results which appear to be new and important, but in which almost everything depends on the precise rigour of the methods of proof which you have used.

As instances of these 3 classes I may mention .....

Ramanujan's second (10-page) letter dated Feb. 27, 1913, was in response to Hardy's reply. In this letter Ramanujan wrote ([XI], p.53):

> *I find in many a place in your letter rigorous proofs are required and so on and you ask me to communicate the methods of proof. I am sure you will follow the London Professor. But as a fact I did not give him any proof but made some assertions as the following under my new theory. I told him that the sum of an infinite number of terms of the series:-* $1+2+3+4+\cdots=-\frac{1}{12}$ *under my theory. If I tell you this you will at once point out to me the lunatic asylum as my goal. I dilate on this simply to convince you that you will not be able to follow my method of proofs if I indicate the lines on which I proceed in a single letter. You may ask how you can accept results based on wrong premises. What I tell you is this. Verify the results I give and if I agree with your results, got by treading the groove in which the present day mathematicians move, you should at least grant me that there may be some truth in my fundamental basis. So what I now want at this stage is for eminent professors like you to recognize that there is some worth in me. I am already a half starving man. To preserve my brains I want food and this is now my first consideration. Any sympathetic letter from you will be helpful to me here to get a scholarship either from the University or from the Government.*
>
> *...You may judge me hard that I am silent on the methods of proof. I have to re-iterate that I may be misunderstood if I give in a short compass the lines on which I proceed. It is not on account of my unwillingness on my part but because I fear I shall not be able to explain everything in a letter. I do not mean that the methods should be buried with me. I shall have them published if my results are recognized by eminent men like you.*

The passages quoted clearly indicate that Ramanujan was fully aware of the apparently absurd nature of the results and also the difficulty he had in an unambiguous presentation of his 'startling' results. In the words of Profs. Bruce C. Berndt and Robert A. Rankin [XI, p.17]:

> *Evidently, Hill shared Abel's view that 'divergent series are in general deadly', for, not surprisingly, he failed to discern the origin of the three results quoted in his letter. The three claims give the values of* $\zeta(-n)$, *for* $n = 1, 2, 3$, *respectively*[7]. *It was natural for Ramanujan to write these facts in terms of the divergent series of the Riemann zeta-function to these values. In Ramanujan's theory of the 'constant' of a (convergent or divergent) series, the three values* $-\frac{1}{12}, 0$ *and* $\frac{1}{240}$ *are, respectively, the constants of the three given series. It is likely that this theory was not communicated to Hill. Ramanujan's theory of divergent series is elucidated, albeit rather imprecisely, in Chapter 6 of his second Notebook and readers should consult Berndt's book (Ramanujan's Notebooks* [VIII], *Part* II*) for an account of this theory.*

The years of fruition

The life of Ramanujan, in the words of C.P. Snow [10] *is an admirable story, and one which showers credit on nearly everyone*[11]. Sir Francis Spring, the Chairman and Mr. S. Narayana Iyer, the Manager of Madras Port Trust gave Ramanujan every possible encouragement. Dr. Gilbert T. Walker, F.R.S., Director General of Observatories, Simla, visited University of Madras in February 1913 and Sir Francis Spring drew his attention to Ramanujan's Notebooks. Dr. Walker, a good mathematician and a Senior Wrangler, was a former Fellow of Trinity College, Cambridge, as well as a lecturer and he immediately recognized the intrinsic quality of Ramanujan's work. He wrote [12] to Mr. Francis Dewsbury, the Registrar of Madras University, commending the work of Ramanujan to be *comparable in originality with that of a Mathematics Fellow in a Cambridge college*, though lacking in the precision and completeness necessary for establishing the universal validity of the results. He wrote that it was perfectly clear to him

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[7]The Riemann zeta-function is defined as: $\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}, \quad Re\ s > 1$. The ζ function has a unique analytic continuation to the point $s = -1$, where we get $\zeta(-1) = -1/12$, which is what Ramanujan writes as: $1 + 2 + 3 + \cdots + \infty = -1/12$. In the 'zeta function regularization' method resorted to by String theorists, this result is used for understanding the necessity of choosing the space-time coordinates to be 26 to ensure the Lorentz invariance of the light cone formalism [80, p.96]. I am thankful to Prof. R. Ramachandran for drawing my attention to this point.

> *that the university would be justified in enabling S. Ramanujan for a few years at least to spend the whole of his time on mathematics without any anxiety as to his livelihood.*

He also wanted the University to correspond with Mr. Hardy, Fellow of Trinity College, Cambridge, since Ramanujan was already in correspondence with Hardy, assuring Mr. Hardy of the University's interest in Ramanujan. The recommendation of Dr. Walker was accepted by the Board of Studies of the University of Madras, and Ramanujan was granted a special research scholarship of Rs.75/- per month[8] for two years with the express consent of the Governor of Madras and the condition that Ramanujan should submit quarterly reports[9] on his work . The Madras Port Trust granted Ramanujan two years leave (on loss of pay) to enable him to accept this scholarship from May 1913, as the **first research scholar of the University of Madras**. Thus began his research career as a professional mathematician.

Ramanujan's first letter [13] to Prof. Hardy, dated 16th January 1913, contained *the bare statements of about 120 theorems, mostly formal identities from the Notebooks.* This collection obviously represented what Ramanujan himself considered were results of importance. Prof. Hardy, the professional mathematician, who was aware that he was *the first really competent person who had the chance to see some of his work*, found some of the series formulae intriguing, some of the integral formulae which were classical and known, vaguely familiar and he could prove some integral formulae with effort but these were to him the least impressive. However, some of Ramanujan's formulae were *on a different level and obviously both difficult and deep* (more on these formulae in Chapter 2). C.P. Snow in his Rectorial Address [14] delivered before the University of St.Andrews, Scotland, on 13th April 1962, records Hardy's reaction to this letter of Ramanujan:

> *Hardy gave the manuscript a perfunctory glance, and went on reading the morning paper. It occurred to him that the first page was a little out of the ordinary for a cranky correspondent. It seemed to consist of some theorems, very strange-looking theorems, without any argument. Hardy then decided that the man*

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[8]Ramanujan was getting a salary of Rs. 25/- per month when he was in the A.G.'s Office and Rs. 30/- p.m. when he was at the Madras Port Trust. Also, The salary of a Professor in those days was Rs. 225/- per month.

[9]For these reports refer [VIII].

> *must be a fraud, and duly went about the day according to his habits, giving a lecture, playing a game of tennis. But there was something nagging at the back of his mind. Anyone who could fake such theorems, right or wrong must be a fraud of genius. Was it more or less likely that there should be a fraud of genius or an unknown Indian mathematician of genius? He went that evening after dinner to argue it out with his collaborator, J.E. Littlewood, whom Hardy always insisted was a better mathematician than himself. They soon had no doubt of the answer. Hardy was seeing the work of someone whom, for natural genius*[10]*, he could not touch – who, in natural genius, though of course not in achievement, as Hardy said later, belonged to the class of Euler and Gauss.*

Hardy made up his mind that Ramanujan should be brought to Cambridge and provided with the necessary education and contact with western mathematicians of the highest class. So, Hardy, wrote to the Secretary of the Indian students, in the India Office, London, suggesting that some means be found to get Ramanujan to Cambridge and he in turn wrote, in February 1913, to Mr. Arthur Davies, the Secretary to the Advisory Committee for Indian students in Madras conveying the desire of the tutors at Trinity to get Ramanujan to Cambridge.

In quick succession, Ramanujan received in the next three months four long letters[11] from Hardy in which the latter wrote plainly about what had been proved or claimed to have been proved by Ramanujan. He clearly communicated his genuine anxiety *to see what can be done to give you (Ramanujan) a better chance of making the best use of your obvious mathematical gifts.* At last Ramanujan had found a sympathetic friend in Hardy and was willing to place unreservedly in his hands all that he had. He wrote again to Hardy on 27th February 1913 and sent him more formulae and explanations. On 17th April 1913, Ramanujan wrote to Hardy about his having secured the scholarship, of £60 per annum, of the University of Madras, for two years. Ramanujan took up residence at Hanumantharayan Koil Lane in Triplicane

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[10]The underlining in this passage is by the author of this work and not by C.P. Snow.

[11]Refer Bruce C. Berndt and Robert A. Rankin : *Ramanujan : Letters and Commentary* [XI], for these and other letters referred to in this Chapter.

around this time and had perhaps access to books on mathematics in the University library.

Ramanujan was initially reluctant to go abroad because of his own caste prejudices[12] in those days which were compounded by the extremely orthodox views of his mother to whom he was greatly attached. At the beginning of 1914, Mr. E.H. Neville, a young mathematician and a Fellow of Trinity College, Cambridge, was in Madras as a visiting lecturer to give a series of lectures on Differential Geometry to Mathematics Honours students of the University of Madras. Mr. Hardy entrusted him with the mission of persuading Ramanujan to visit Cambridge. Mr. Neville met Ramanujan and saw his priceless notebooks. This was sufficient to convince him of Ramanujan's uncommon ability and to make him take over the initiative to overcome all the difficulties in arranging for Ramanujan's visit to Cambridge. Prof. Richard Littlehails, who was a Professor of Mathematics with the observatory in Madras introduced Neville [15]

> *to everyone who carried weight in the University or in the civil administration; everywhere I talked of Ramanujan, explained as I have tried to do now the importance to him of a stay in Cambridge, and urged generosity.*

In a letter [16], dated 28th January 1914, to Mr. Dewsbury, the Registrar of the University of Madras, Mr. Neville wrote about

> *the importance of securing to Ramanujan a training in the refinements of modern methods and a contact with men who know what range of ideas have been explored and what have not*

and prophesied that Ramanujan would respond to such a stimulus and that

> *his name will become one of the greatest in the history of mathematics, and the University and city of Madras will be proud to have assisted in his passage from obscurity to fame.*

The very next day, Prof. Littlehails also wrote [17] to Mr. Dewsbury that Ramanujan

> *be granted by this University a scholarship of about £250 (Sterling) together with a grant of about £100 in order to enable him*

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[12]Crossing the oceans was considered a sacrilege by the Hindu Brahmins and often people who did so were, on their return to India, treated as outcastes. All relationships with the individual and even their families were shunned!

> *to proceed to Cambridge. Ramanujan is a man of most remarkable mathematical ability, amounting I might say to genius, whose light is metaphorically hidden under a bushel in Madras.*

They wrote these letters after Ramanujan was persuaded to put aside his prejudices against crossing the sea. Hardy stated that Ramanujan's

> *consent was at last got very easily....(when) his mother announced that she had a dream in which she saw her son seated in a big hall amidst a group of Europeans and that the goddess Namagiri had commanded her not to stand in the way of her son fulfilling his life's purpose.*

The proposals regarding the scholarship to be granted to Ramanujan by the University of Madras were approved. To the lasting credit of the University of Madras, the Syndicate decided within a week to set aside Rs.10,000/- to offer Ramanujan a scholarship of £250 a year plus £100 for a passage by ship and for initial outfit[13]. At the instance of Professors Neville and Littlehails, Sir Francis Spring wrote [18] to the personal Secretary (Mr. C.B. Cotterell) to the Governor (Lord Pentland) of Madras, persuading His Excellency to speedily approve the University's sanction. Government sanction too was granted within a week.

It is remarkable that while there was no one in the Colleges or the University who could appreciate the depth of Ramanujan's mathematical abilities, the University of Madras awarded its first research scholarship to Ramanujan to study in Cambridge, despite the fact he had failed twice to pass the First degree in Arts (F.A.) examinations of the University. This is undoubtedly due to the courage of conviction of Indian stalwarts of that era – Dewanbahadur Ramachandra Row, V. Ramaswamy Iyer, S. Narayanan Iyer and P.V. Seshu Aiyar – who succeeded in convincing and persuading the British authorities to act immediately on the basis of the merits of the candidate.

This offer of the University of Madras was made to Ramanujan in February 1914. He sent his wife and mother back to Kumbakonam, changed the tra-

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[13]The second class fare between London and Bombay was £32 in 1914, or about Rs. 480. (*British Passenger Liners of the Five Oceans*, by C.R. Vernon Gibbs (London: Putnam, 1963), 63. cf.[X], p.397).

ditional hair-style of the brahmin, viz. a tuft, and got his hair trimmed in European style and left Madras by S.S. Nevasa on 17th March 1914. Prior to his departure, he arranged with the University for £60 a year to be sent to his parents in Kumbakonam, out of his annual scholarship amount. Mr. Arthur Davies and Prof. Littlehails attended to all the details regarding Ramanujan's passage to England. Except for the first three days when he was sea-sick, Ramanujan enjoyed the voyage and reached London through the Channel and the Thames on 14th April 1914. He was received by Mr. E.H. Neville and his brother at the docks and stayed at Cromwell Road for a few days before going to Cambridge on the 18th evening. He remained for a few days in Mr. Neville's house before moving to the college premises for stay, which even though costlier than lodging houses, was more convenient for him and the professors. Ramanujan wrote [19] to his friend that

> *Mr. Hardy, Mr. Neville and others here are unassuming, kind and obliging. As soon as I came here, Mr. Hardy paid £20 to the college for my entrance and other fees and made arrangements to give me a scholarship of £40 a year.*

Ramanujan was admitted by Mr. Hardy to Trinity College which supplemented his scholarship with the award of an exhibition of £60 a year, to augment the £250 a year scholarship awarded by the University of Madras.

Though Ramanujan had access only to Carr's Synopsis, – and perhaps, to a few other books[14] – still, in the words of the historian J.R. Newman [20], he

> *arrived in England abreast and often ahead of contemporary mathematical knowledge. Thus, in a lone mighty sweep, he had succeeded in recreating in his field, through his own unaided powers, a rich half century of European mathematics. One may doubt whether so prodigious a feat had ever before been accomplished in the history of thought.*

To Mr. Hardy [21] Ramanujan's friend, philosopher and discoverer:

> *The limitation of his knowledge was as startling as its profoundity. Here was a man who could work out modular equations, and theo-*

[14] From the article of Mr. Narayana Iyer's son [I, p.112], as stated earlier, Ramanujan had access to a book on Jacobi's elliptic functions. Unfortunately, it is not possible to ascertain, from the records of the Library of the University of Madras, what books were available for reference to Ramanujan.

> *rems of complex multiplications, to orders unheard of, whose mastery of continued fractions was, on the formal side at any rate, beyond that of any mathematician in the world, who had found for himself the functional equation of the zeta-function, and the dominant terms of many of the most famous problems in the analytic theory of numbers, and he had never heard of a doubly periodic function or of Cauchy's theorem, and had indeed but the vaguest idea of what a function of a complex variable was. His ideas of what constituted a mathematical proof were of the most shadowy description. All his results, new or old, right or wrong, had been arrived at by a process of mingled argument, intuition and induction, of which he was entirely unable to give a coherent account.*

With such a natural genius, Hardy collaborated and tried to teach, as he wrote, the

> *things of which it was impossible that he should remain in ignorance. ... It was impossible to allow him to go through life supposing that all the zeroes of the zeta function were real. So I had to try to teach him, and in a measure I succeeded, though I obviously learnt from him much more than he learnt from me.* (cf. [X], p.226).

Hardy did not attempt to convert Ramanujan into a mathematician of the modern school but enabled him to go on producing original ideas in his classical mould with rigorous proofs for the theorems he discovered.

The period of Ramanujan's stay in England almost overlapped with the years in which World War I took place. *One of the lecturers went to war*[15], wrote Ramanujan [22] to a friend in India and Ramanujan felt that *the other professors ...lost their interest owing to the... war.* One of the professors had remarked that Ramanujan was in England at the most unfortunate time. There were about 700 students before the war, but this number was reduced to 150 by November 1915.

Initially he asked for and obtained some South Indian food items (like tamarind, coconut oil, etc.) by post parcel from his home, as well as from a

[15]Ramanujan was perhaps referring to the departure of Mr. J.E. Littlewood.

company in London but by January 1915, he wrote [23] to a friend of his in India that

> *now as well as in the future I am not in need of anything as I gained control over my taste and can live on mere rice with a little salt and lemon juice for an indefinite time.*

His *difficulty of getting proper food* was alleviated by the availability of good milk and fruits. Being a vegetarian he had no option but to cook for himself.

He was attending a lecture by Mr. Berry at the University on elliptic integrals. Mr. Berry was working out some formulae on the black-board and a glance at Ramanujan's face, alight with excitement, caused him to ask Ramanujan whether he was following the lecture and whether he had anything to say. At this Ramanujan went to the black-board and much to everyone's surprise wrote down some of the results which were yet to be proved. This anecdote was recalled by P.C. Mahalanobis [24], the eminent Indian statistician.

Ramanujan wrote a few articles soon after he reached Cambridge and in June 1914, Hardy presented some of the results from Ramanujan's Notebooks at a meeting of the London Mathematical Society. However, in January 1915, Ramanujan wrote [23] to a friend in India that his *notebook is sleeping in a corner for these four or five months.* Ramanujan was more interested in getting new results (and partly due to the ongoing war), he decided to publish the old results worked out in his Notebooks after the war. After about a year and a half at Cambridge, Hardy wrote to the Registrar of the University of Madras, that Ramanujan

> *is beyond question the best Indian mathematician of modern times .... He will always be rather eccentric in his choice of subjects and methods of dealing with them .... But of his extraordinary gifts there can be no questions; in some ways he is the most remarkable mathematician I have ever known.*

Hardy's letter [25] and official report to the University, as well as an appeal by Sir Francis Spring to the University to continue the assistance extended by it to Ramanujan, made the University (in December 1915) extend the scholarship up to March 1919.

During his five year stay in Cambridge, Ramanujan published twenty one research papers containing theorems on definite integrals, modular equations, Riemann's zeta function, infinite series, summation of series, analytic number theory, asymptotic formulae, modular functions, partitions and combinatorial analysis. His paper entitled *Highly Composite Numbers* which appeared in the Journal of the London Mathematical Society, in 1915, is 62 pages long and contains 269 equations. This is his longest paper. The London Mathematical Society had some financial difficulties at that time and Ramanujan was requested to reduce the length of his paper to save printing expenses. Five of these 21 research papers were in collaboration with Hardy. Ramanujan also published five short notes in the Records of Proceedings at meetings of the London Mathematical Society and six more in the journal of the Indian Mathematical Society.

The *Collected Papers of Srinivasa Ramanujan* [III], published in a volume running to 350 pages, was edited by G.H. Hardy, P.V. Seshu Aiyar and B.M. Wilson. The first edition of this book, published by Cambridge University Press, in 1927, resulted in a flurry of research papers (ref.[VIII]) by G.N. Watson (25), C.T. Preece (6), W.N. Bailey (4), Robert A. Rankin (3) and some others during the period 1928 – 38.

Ramanujan was awarded the B.A. degree by research in March 1916 for his work on *Highly composite numbers* and published as a long paper. Ramanujan's dissertation bore the same title and included six other papers. Ramanujan was registered as a research student in June 1914 and the prerequisite of a diploma or a certificate, as well as the domiciliary requirement of six terms must have been relaxed in his extraordinary case. It is unfortunate that a copy of this dissertation is not to be found in the records of the University ([XI], 137). According to Hardy ([III], 499), this work of Ramanujan

> *is a very peculiar one, standing somewhat apart from the main channels of mathematical research. But there can be no question as to the extraordinary insight and ingenuity which he has shown in treating it, nor any doubt that the memoir is one of the most remarkable published in England for many years.*

Ramanujan's designated tutor who monitored his progress at Trinity College, Cambridge, was E.W. Barnes, who considered Ramanujan as perhaps the most brilliant of all the top Trinity students, which included Littlewood.

([X], 233). Hardy was immensely satisfied with the progress of Ramanujan and wrote so to the Registrar of the University of Madras supporting an extension of Ramanujan's two-year scholarship

> *... until, as I confidently expect, he is elected to a Fellowship at the College. Such an election I should expect in October 1917.*

Later, in June 1916, in an official report on the progress of Ramanujan's work in England to the University's Registrar, he wrote:

> *... it is already safe to say that Mr. Ramanujan has justified abundantly all the hopes that were based upon his work in India, and has shown that he possesses powers as remarkable in their way as those of any living mathematician. ··· I have said enough, I hope, to give some idea of his astonishing individuality and power. India has produced many talented mathematicians in recent years, a number of whom have come to Cambridge and attained high academical distinction. They will be the first to recognize that Mr. Ramanujan's work is of a different category.*

In spite of the war which was raging, which deprived Ramanujan of the center stage which he would otherwise have held with his brilliant research work in the midst of his peers, the confidence he kindled in Hardy was enough to win for him recognition and laurels very soon, but, unfortunately, the first signs of illness appeared in Ramanujan in the spring of 1917.

Ramanujan's medicography

Ramanujan's biographer, Ranganathan [VI], stated that by the end of 1918, it was definitely known that tuberculosis had set in. In the more recent and complete biography, Kanigel [X], also lays emphasis on tuberculosis as being the cause of Ramanujan's prolonged, terminal illness. A systematic study of the details led Rankin [93], in 1984, to point out that Ramanujan's illness was not properly diagnosed and tuberculosis was not the cause of his death. It is a fact that the warmer climates of Madras and its surroundings did not show any marked improvement in his health after his return. On the contrary, his condition further deteriorated and towards the end, he was reduced to 'only skin and bone', as described by his wife.

Young [94], in 1994, has researched into the illness of Ramanujan, more recently, and this 'latest detective work' gives us a better insight into the health

of Ramanujan. We give here a brief summary of this medical biography (or, shall we coin the word *medicography*!):

Ramanujan came to the city of Madras, in 1906, and joined the Pachaiyappa's College to study in the F.A. class again. But, a few months later, he fell ill with dysentery and had to return to Kumbakonam for a period of about three months. It is conjectured [86], after sieving through all the information, that Ramanujan's dysentery was caused by amoebiasis, a tropical infection widespread in the metropolitan cities of India.

> *Amoebiasis, unless adequately treated, is a permanent infection, although many patients may go for long periods with no overt signs of the disease. Relapses occur when the host-parasite relationship is disturbed. Ramanujan experienced such a relapse, I believe, in 1909 when according to his friend R.R. Ayyar [VI]:*
>
> > *Ramanujan, who was living in [Madras], became seriously ill .... As a patient ... he was obstinate and would not drink hot water and insisted on eating grapes which were sour and bad for him. [The doctor] after examining him, asked me to send him to his parents as his condition required constant nursing.*
>
> *How ill Ramanujan felt at this time is indicated by his giving his host for safe keeping the two large notebooks kept with him all the time and in which he had been recording his mathematical results; the same notebooks that are now famous as a major legacy of his genius.*
>
> *Later the same year (1909), while still at home with his family, he developed a hydrocele, which was operated on in January 1910 [X]. ... Dr. Shaw's suspicion that the operation was the excision of a malignant growth, depending as it must have done on Ramanujan's exact description of the lesion, certainly favours the explanation of a scrotal amoeboma rather than a hydrocele.*

These were the illnesses suffered by Ramanujan before and after his marriage to Janaki (1909) and before his departure to England. From all accounts available it appears that his health was reasonably good during the

first three years of his stay in Cambridge, despite his strict vegetarian diet, the food shortages and his own 'cooking only once a day or two'. This was also his most productive period in Cambridge. From May 1917, when he was first admitted to the Nursing Hostel in Cambridge for 5 months, he seems to have been in and out of TB Sanatoria – Mendip Hills in Somerset (2-3 weeks in Oct. 1917), Matlock House in Derbyshire (Nov. 1917 - June 1918), Fitzroy House in London (June - Dec. 1918) and Colinette House, Putney (end of Dec. 1918) – until his departure to India in March 1919.

Symptoms of improvement showed, after considerable treatment for tuberculosis, in the autumn of 1918. He was able to meet all his medical expenses incurred during his illness out of his earnings accumulated through frugal living. Thanks to the unstinted efforts of Hardy, who did his best to get Ramanujan due recognition, he was elected a Fellow of the Royal Society of London in February 1918. Reproduced here, on p.27, is an excerpt from the Records of the Royal Society, dated December 18, 1917 (a copy of this document is an exhibit in the Ramanujan Museum in Royapuram, Madras).

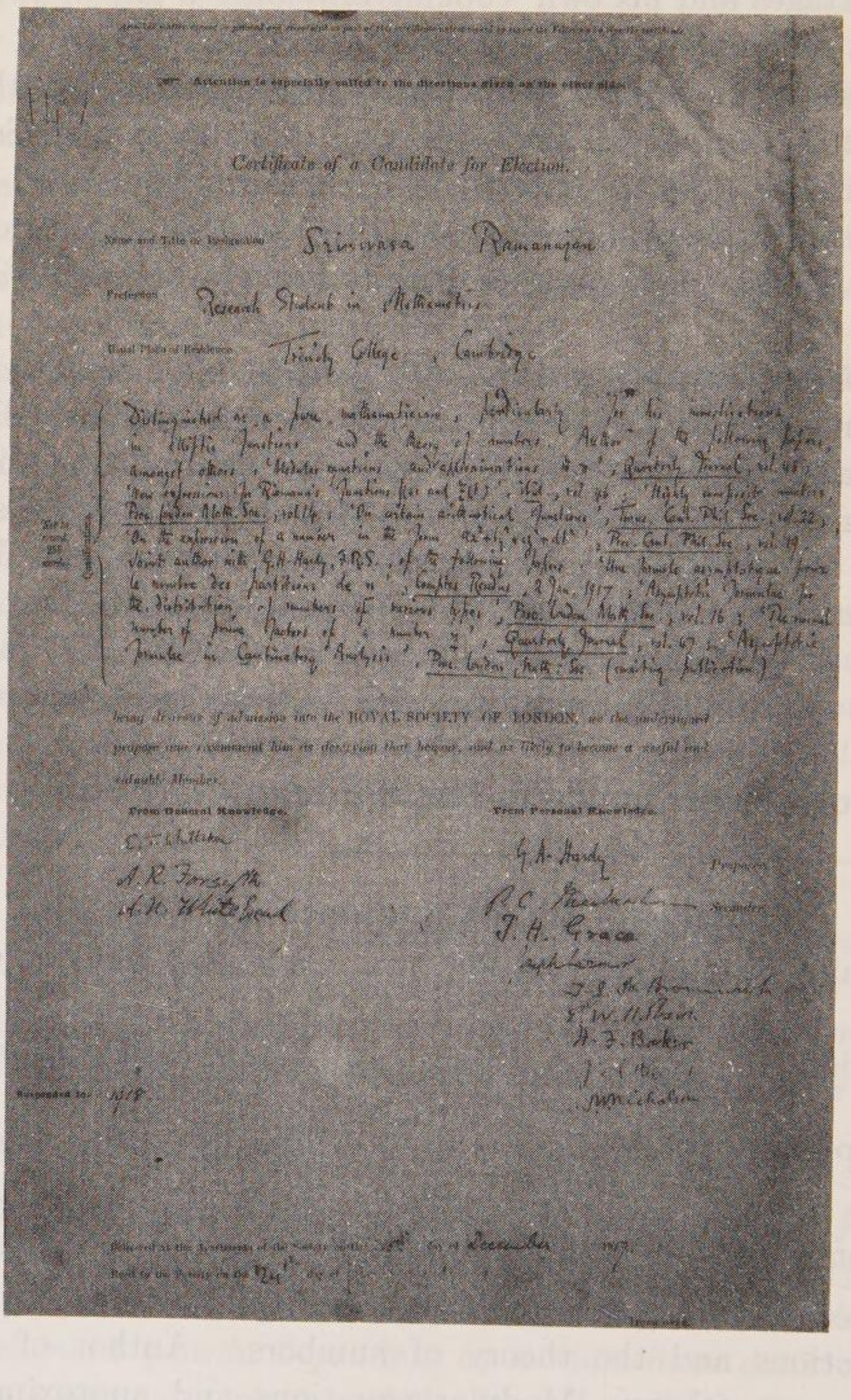
Excerpt from Records of the Royal Society & F.R.S.

Certificate for a Candidate for Election.

Name and Title or Designation	*Srinivasa Ramanujan*
Profession	*Research Student in Mathematics*
Usual Place of Residence	*Trinity College, Cambridge*

Qualifications(Not to exceed 250 words):

Distinguished as pure mathematician, particularly for his investigations in elliptic functions and the theory of numbers. Author of the following papers, amongst others: 'Modular equations and approximations to π', Quarterly Journal, vol. 45; 'New expressions of Riemann's function $\zeta(s)$ and $\Xi(t)$', ibid, vol. 46; 'Highly composite numbers', Proc. London. Math. Soc., vol. 14; 'On certain arithmetical functions', Trans. Camb. Phil. Soc., vol. 22; 'On the expression of a number in the form $ax^2 + by^2 + cz^2 + dt^2$,' Proc. Camb. Phil. Soc., vol. 19.



Certificate of a Candidate for Election.

Name and Title of Candidate: Srinivasa Ramanujan

Profession: Research Student in Mathematics

Usual Place of Residence: Trinity College, Cambridge

Distinguished as a pure mathematician, particularly for his investigations in elliptic functions and the theory of numbers. Author of the following papers, amongst others: 'Modular equations and approximations to π', Quarterly Journal, vol. 45; 'New expressions for Riemann's functions $\xi(s)$ and $\Xi(t)$', ibid., vol. 46; 'Highly composite numbers', Proc. London Math. Soc., vol. 14; 'On certain arithmetical functions', Trans. Cam. Phil. Soc., vol. 22; 'On the expression of a number in the form $ax^2+by^2+cz^2+du^2$', Proc. Cam. Phil. Soc., vol. 19; (with author with G.H. Hardy, F.R.S.) of the following papers: 'Une formule asymptotique pour le nombre des partitions de n', Comptes Rendus, 2 Jan. 1917; 'Asymptotic formulae in Combinatory Analysis', Proc. London Math. Soc., vol. 16; 'Asymptotic formulae for the distribution of integers of various types', Proc. London Math. Soc., vol. 16; 'The normal number of prime factors of a number n', Quarterly Journal, vol. 47; 'Asymptotic formulae in Combinatory Analysis', Proc. London Math. Soc. (now in course of publication).

Being desirous of admission into the ROYAL SOCIETY OF LONDON, we the undersigned propose and recommend him as deserving that honour, and as likely to become a useful and valuable Member.

From General Knowledge.

C. Smith (Master)
A.R. Forsyth
A.N. Whitehead

From Personal Knowledge.

G.H. Hardy — Proposer
P.A. MacMahon — Seconder
J.H. Grace
Sept. Lovett
J.J. Thomson
E.W. Hobson
H.F. Baker
J.C. [illegible]
W.H. Eccles

Recommended by: H.F.B.

Balloted at the Meeting of the Society on the 18th day of December 1913.

Read to the Society on the 22nd day of [illegible]

Fig. caption: Note that the proposer is G.H. Hardy and the seconder P.A. MacMahon. It is interesting to note that among those who were signatories with 'Personal knowledge' of Ramanujan were E.W. Hobson and H.F. Baker, *who had not replied to letters written by Ramanujan from India.* ([XI], 155.)

Joint author with G.H. Hardy, F.R.S., of the following papers: 'Une formulae asymptotique pour le nombre des partitions de n', Comptes Rendus, 2 Jan. 1917; 'Asymptotic Formulae for the distribution of numbers of various types', Proc. London Math. Soc., vol. 16; 'The normal number of prime factors of a number n', Quarterly Journal, vol. 47; 'Asymptotic Formulae in Combinatory Analysis', Proc. London Math. Soc., (awaiting publication).

being desirous of admission into the ROYAL SOCIETY OF LONDON, *we the undersigned propose and recommend him as deserving that honour, and as likely to become a useful and valuable Member.*

From General Knowledge.	**From Personal knowledge.**	
(sd.) E.T. Whittaker	(sd.)G.H. Hardy	*Proposer.*
(sd.) A.R. Forsyth	(sd.) P.A. MacMahon	*Seconder.*
(sd.) A.N. Whitehead	(sd.) J.H. Grace	
	(sd.) Joseph Larmor	
	(sd.) T.J.I'A. Bromwich	
	(sd.) E.W. Hobson	
	(sd.) H.F. Baker	
	(sd.) J.E. Littlewood	
	(sd.) J.W. Nicholson	

Suspended for 1918.

Delivered at the Apartments of the Society on the 18th day of Dec. 1917.
Read to the Society on the 24th day January 1918.

This certificate on a printed form of the Royal Society has been filled by hand (and the hand writing appears to be that of Mr. Hardy). The citation awarding the Fellowship of Royal Society to Ramanujan read:

> *Srinivasa Ramanujan, Trinity College, Cambridge. Research student in Mathematics Distinguished as a pure mathematician particularly for his investigations in elliptic functions and the theory of numbers.*

The renowned Physicist, Prof. R.H. Dalitz (Oxford University), on his visit to the Institute of Mathematical Sciences, in Feb. 1996, spent some time with me discussing various aspects concerning Raman and Ramanujan the

two giants of Indian science, and Chandra who was a personal friend of Prof. Dalitz and with whom I have had some meetings and correspondence. After Prof. Dalitz returned to Oxford, we had several interesting exchanges by e-mail and one such is regarding Ramanujan's F.R.S.[16]:

> *I did not succeed in finding Ramanujan's signature in the Royal Society book. The book is indexed, so it is just not there. The reason undoubtedly is that he was ill in that period and could not go to the Royal Society to sign it. There are other examples of well-known FRS's who somehow didn't get their signature into the book. That means that he did not ever attend any meeting of the Royal Society; if he had, they would have brought out the book and not let him go until he had signed. Of course, it was also war-time, which meant that there were as few meetings as possible.*

Ramanujan was elected to a Trinity College Fellowship, in October 1918, which was a Prize Fellowship worth £250 a year for six years with no duties or conditions. These awards acted as great incentives to Ramanujan who discovered some of the most beautiful theorems in mathematics, subsequently.

Hardy's letter to the Registrar of the University of Madras, Mr. Dewsbury, dated Nov. 26, 1918 ([XI], p.199) struck a hopeful note:

> *There is at last, I am profoundly glad to say, a quite definite change for the better. I think we may now hope that he has turned the corner, and is on the road to recovery. His temperature has ceased to be irregular, and he has gained nearly a stone*[17] *in weight. The consensus of medical opinion is that he has been suffering from some obscure source of blood poisoning, which has now dried up; and that it is reasonable to expect him to recover his health completely and if all goes well fairly rapidly.*

Ramanujan's symptoms were predominantly night-time fever, loss of weight leading to his emaciated looks and these caused depressions which once drove him to the limit of attempting suicide (a story which was recounted many years after his death, by Dr. S. Chandrasekhar as told to him by Prof. Hardy, and reproduced in chapter 5 of this book). These symptoms made the doc-

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[16] Private communication, by e-mail, dated 29 March 1996.

[17] One Stone weight is equal to 14 pounds.

tors consider various diagnosis, at different times: gastric ulcer, malaria, tuberculosis, cancer of the liver, etc. In recent times, with hind sight, vitamin B_{12} deficiency (something unknown to the world at that time) has been diagnosed as a possibility [85].

The recovery alluded to by Hardy in his letter to Dewsbury was obviously the reason why Ramanujan was persuaded to return to India, with the hope that he would soon recover and return to take up the Trinity College Fellowship awarded to him for five years.

The beginning of the end

After completing nearly five years at Cambridge, early in 1919, when Ramanujan appeared to have recovered sufficiently to withstand the rigours of a long voyage to India, he left England on 27th February 1919 by S.S. Nagoya. Four weeks later on 27th March he arrived at Bombay and soon after at Madras, thin, pale and emaciated, but *with a scientific standing and reputation such as no Indian has enjoyed before.* Professor Hardy who expressed this view [25] also hoped that *India will regard him as the treasure he is.* He urged the Madras University to make a permanent provision for him to enable him to continue his research work. Again the University rose to the occasion by granting Ramanujan £250 a year as an allowance for five years, commencing from April 1919. He was sent back to India by Hardy with the fond hope that the warmer climate would help complete his recovery from a tubercular tendency.

Most unfortunately his precarious health did not improve, on his return to India. Fevers relapsed and in addition, his wife recalled that he suffered severe bouts of stomach pain too [VI]. Ramanujan was subject to fits of depression, had a premonition of his death and was a difficult patient. He spent 3 months in Madras, 2 months in Kodumudi and 4 months at Kumbakonam. When his condition showed signs of further deterioration, after great persuasion, Ramanujan was brought to Madras for expert medical treatment, in January 1920. Despite all the tender attention he could get from his wife who nursed him throughout this period, and the best medical attention from the doctors, his untimely end came on 26th April 1920, at Chetput, Madras, when Ramanujan was 32 years, 4 months and 4 days old. His wife lived with him,

after she came of age, only for a year before his departure to England, and looked after him during his illness after his return. Even during those months of prolonged illness he kept on working, though in a reclining position, at a furious pace and kept jotting down his results on sheets of paper. In his last and only letter to Hardy written after his return to India, in January 1920, Ramanujan communicated his original work on what he called *mock theta functions.*

From the available evidence and retrospective diagnosis, Young [94] makes out the case for *hepatic amoebiasis*, a tropical disease contacted by Ramanujan in 1906, as the cause of his terminal illness. His reason as to why this was not recognized at that time is best recounted in his own words:

> *Hepatic amoebiasis was regarded in 1918 as a tropical disease ('tropical liver abscess'), and this would have had important implications for successful diagnosis, especially in provincial medical centers. Furthermore, the specialists called in were experts in either tuberculosis or gastric medicine. Another major difficulty is that a patient with this disease would not, unless specifically asked, recall as relevant that he had had two episodes of dysentery 11 and 8 years before. Finally, there is the very good reason that, because of the great variability in physical findings, the diagnosis was difficult in 1918 and remains so today: hepatic amoebiasis 'presents a severe challenge to the diagnostic skills . . . [and] should be considered in any patient with fever and an abnormal abdominal examination coming from an endemic area'.*

Hardy, who was unaware that the end was to come so soon was shocked when it came prematurely. He was of the view that *a mathematician is often comparatively old at 30.* For, in his roll-call of mathematicians, Hardy wrote ([XII], p.71):

> *Galois died at twenty-one, Abel at twenty-seven, Ramanujan at thirty-three, Riemann at forty . . . I do not know an instance of a major mathematical advance initiated by a man past fifty*[18].

[18] A few examples which can be cited which explode *The Myth of the Young Mathematician* are: Newton's *Principia* was written when he was in his mid 40s; Euler, despite his blindness, produced his three volumes on integral calculus when he was in his 60s; Gauss at 34 proposed his theory of analytic functions; and in more recent times, Cartan,

Ramanujan's wife: Janakiammal (Janaki)

Ramanujan got married, on July 14, 1909, to the then nine-year old Janaki, the fourth of six children (including a son), born on March 21, 1899, and daughter of Rangaswamy Iyengar and Ranganayaki Ammal of Rajendram, a village close to Marudur Railway Station. The young wife came of age and joined her husband in 1912, after Ramanujan got a clerical job in the Madras Port Trust on a monthly salary of Rs.30. They set up house in Saiva Muthiah (Mudali) Street[19], George Town. In May 1913, Ramanujan joined the University of Madras as its first research scholar and he first moved, for about 10 months, to Hanumantharayan Koil Street, and then to Toppu Venkatachala (Mudali) Street, both in Triplicane. His wife and mother lived with him for some months, at the latter residence, before he left for England, on March 17, 1914.

Ramanujan was alone in England for almost five years (April 14, 1914, to Feb.27, 1919) and he fell ill in the spring of 1917, as stated earlier. The ongoing first World War prevented his early return to India and the possibility of his wife joining him to take care of him. When Ramanujan returned to India, in April 1919, emaciated from suspected tuberculosis but with a great reputation, Janaki joined him in Madras and nursed him till his untimely death on April 26, 1920. During those months, the household was run by Ramanujan's mother Komalathammal and grandmother Rangammal. In later years, after Ramanujan's death, Janaki was happy to state:

> *I considered it my good fortune to give him rice, lemon juice, butter milk, etc., at regular intervals and to give fomentation to his legs and chest when he reported pain. The two vessels used then*

Poincaré, Siegel, Kolmogorov and Erdös exhibited creativity in mathematics in their later years. (cf. Susan Landau, Notices of the AMS **44** (1997) 1284).

[19]The street names in Madras (officially renamed Chennai in 1996) have been renamed by caste conscious State Government regimes. So ridiculous were some of the changes that N.S.C. Bose Road, Krishnamachari Road and Dr. Nair Road became, respectively, N.S.C. Road, Krishnama Road and Doctor Road! Needless to say, the Government turned a deaf ear to appeals, in 1987, to rename a stretch of Wallajah Road, on which the Ramanujan Institute for Advanced Study in Mathematics of the University of Madras is situated, as Ramanujan Road.

for preparing hot water are alone still with me; these remind me often of those days.

After April 1920, the young widow spent the next eight years of her life in Bombay[20] with her brother, R.S. Iyengar, Assistant Commissioner of Income Tax. During that period, she learnt tailoring and English at home. In 1931, without any resources – having been deprived of all her *sthree dhanam* (dowry) – she returned to Madras and spent a year with one of her sisters and another year with friends, before deciding to live independently. She started to teach tailoring and live in a small house in Hanumantharayan Koil Street, having one room upstairs and one room downstairs, for the next five decades. She earned enough from her tailoring and could even save a little, thanks to her frugal living. In 1950, one of her friends, Soundaravalli, died suddenly entrusting her with her 7 year-old son, W. Narayanan.

Janakiammal took up the responsibility of bringing up this boy and became a foster mother to him. During his school years, she sent him for some years (1952–55) to the Ramakrishna Mission Boarding School. She educated him up to the Bachelors Degree in Commerce (B.Com.) of the University of Madras, in Vivekananda College, Madras, and he became an officer of the State Bank of India. Janakiammal conducted the marriage of her foster son in 1972. The daughter-in-law Vaidehi also became an employee of the State Bank of India. Mr. Narayanan resisted transfers and took voluntary retirement from the Bank in 1988, about 6 years before Janakiammal passed away, to take care of her health. The Narayanans have a son and two daughters and the family took good care of Janakiammal and they were an integral part of her family.

Janakiammal being extremely orthodox and pious, preferred to stay alone in her house in Muthiah (Mudali) II Street, Triplicane, under the loving and tender care of the Narayanans, despite her failing health, till a few years before her death, when she moved to 14, Hanumantharayan Koil Street, also

[20]Bombay has been recently renamed as *Mumbai*.

Fig. caption: Above, Mrs. Janakiammal with her adopted son Mr. W. Narayanan and his wife Mrs. Vaidehi Narayanan. (Photo taken on Aug. 13, 1987, in the seminar hall of the Institute of Mathematical Sciences, Madras, when Mrs. Ramanujan was honoured by the Institute and the Hinduja Foundation.)

in Triplicane. Today, the Narayanans are living in the same house with their three children – Mr. N. Sridhar, C.A., I.C.W.A., Ms. N. Sripriya, M.C.A. and Ms. N. Sri Vidya, XI Standard.

Janakiammal being philanthropic and sympathetic by nature, supported financially the education of several children and other youngsters. She once told the author that some parents used to go to her for token amounts towards their children's examination fees, since they considered getting money from her hands, to be very auspicious.

Janakiammal was receiving a pension from the University of Madras since 1920, the year of her husband's death. This amount was then only Rs.20 per month and it gradually rose to about Rs. 500/- per month, at the time of her passing away. In interviews which appeared in newspapers, soon after

the "Lost" Note Book of Ramanujan was discovered by George Andrews in 1976, Janakiammal lamented the fact that a statue of Ramanujan had never been made, although one had been promised. Prof. Richard Askey saw these newspaper interviews and decided that a bust of Ramanujan was long overdue since Ramanujan's widow wanted one and that

> *In Ramanujan's case a permanent memorial is appropriate: one which can be appreciated by those who do not understand his mathematics should be added to the memorial Ramanujan made for himself with his work.*

So, garnering the support of a hundred mathematicians, including Profs. George Andrews, Bruce Berndt and S. Chandrasekhar, Prof. Askey commissioned a bronze bust, from the only authentic passport size photograph of Ramanujan, by Paul Granlund, sculptor-in-residence at Gustavus Adolphus College at Saint Peter, Minnesota in U.S.A. A copy of the bust was presented to Mrs. Ramanujan in 1985 at a simple function in the University of Madras.

Janakiammal had cherished memories of her great husband. Even in recent years, though she was hard of hearing, had failing eyesight and was having indifferent health, when mathematicians like Profs. Andrews, Askey, Berndt, Bollobas and others[21] made it a point to call on her at her residence, she recalled with pride the prophetic words of her husband that his mathematics would provide for her whether he is alive or dead. She also pointed out on such occasions, when asked, that recognition and financial support to her started coming after the 75th birth anniversary celebrations of Ramanujan in 1962, the year in which a commemorative stamp was brought out by the Government of India. During the centenary year of the Madras Port Trust, she was honoured and was given a pension for life. Since then, the governments of Tamil Nadu, Andhra Pradesh and West Bengal, the Indian National Science Academy, the Ramanujan Mathematical Society, founded in 1985, and the Hinduja Foundation[22] were also giving her monthly pensions.

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[21]The author accompanied some of them to see the gracious old lady. On one such occasion, when a visitor asked her whether she had conjugal relationships with her husband, her face beamed and she answered in the affirmative.

[22]Mr. A.P. Venkateswaran, the Secretary of the Foundation, at the instance of Prof. E.C.G. Sudarshan, the then Director of the Institute of Mathematical Sciences, Madras, was instrumental in presenting a purse of Rs. 20,000/- to Janakiammal, from the funds of

Mrs. Janakiammal Ramanujan, the wife of Srinivasa Ramanujan, acclaimed as the greatest Mathematician of this century, breathed her last on the morning of April 13, 1994, at her residence (14, Hanumantharayan Koil Street, Triplicane, Madras -600 005), at the age of 94.

One of her cherished hopes was that a fitting memorial would come up to honour the man who for natural genius has been compared to all-time greats like, Euler, Gauss and Jacobi, to house the correspondence, papers, and the original Note Books of Srinivasa Ramanujan, besides the large size slate used by him and other such memorabilia. The receptive ears of Prof. Askey resulted in the bust of Ramanujan being made and a copy delivered to her when she was alive. I wonder whether any action will now be taken to build an appropriate memorial, a hope shared by many mathematicians and admirers of Ramanujan, including the present author!

Human qualities

> *In figure he (Ramanujan) was a little below medium height (5 ft. 5 in.) and stout until emaciated by disease; he had a big head, with long black hair brushed sideways above a big forehead; his face was square, he was clean shaven, and his complexion never really dark, grew paler during his life in England; his ears were small, his nose broad, and always his shining eyes were the conspicuous feature that Ramachandra Rao observed in 1910. He walked stiffly, with head erect and toes out-turned; if he was not talking as he walked, his arms were held clear of the body, with hands open and palms downwards. But when he talked, whether he was walking or standing, sitting or lying down, his slender fingers were for ever alive, as eloquent as his countenance.*

This is the physical description of Ramanujan as recorded by Prof. E.H. Neville [15]. Ramanujan had only one passion in life – mathematics. He devoted all his time to this subject and its development. Quoting

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the Hinduja Foundation. An honorarium of Rs.1000/- per month was also announced, at this function at the Institute's premises, presided over by its Patron, Mr. C. Subramaniam, and organized by the author, on Aug. 13, 1987.

Prof. Neville again [15], Ramanujan had

> *an instinctive perfection of manners that made him a delightful guest or companion. Success and fame left his natural simplicity quite untouched. To his friends he was devoted beyond measure, and he devised curiously personal ways of showing his gratitude and expressing his affection. The wonderful mathematician was indeed a lovable man.*

This is in complete accord with the views of Hardy [IV] on Ramanujan:

> *... the picture I want to present to you is that of a man who had his peculiarities like other distinguished men, but a man in whose society one could take pleasure, with whom one could drink tea and discuss politics or mathematics; the picture in short, not of a wonder from the East, or an inspired idiot, or a psychological fraud, but of a rational human being who happened to be a great mathematician.*

The integrity of Ramanujan is transparent from the following statement of Hardy [26]:

> *All of Ramanujan's manuscripts passed through my hands, and I edited them very carefully for publication. The earlier ones I wrote completely. I had no share of any kind in the results, except of course when I was actually a collaborator, or when explicit acknowledgement was made. Ramanujan was almost absurdly scrupulous in his desire to acknowledge the slightest help.*

In a letter to a friend of Ramanujan, in September 1917, Hardy wrote [28]:

> *He has been seriously ill but is now a good deal better. It is very difficult to get him to take proper care of himself; if he would only do so we should have every hope that he would be quite well again before very long.*

In this letter Hardy referred to his discovering that Ramanujan

> *was not writing to his people nor apparently hearing from them. He was very reserved about it and it appeared to us that there must have been some quarrel.*

He expressed his anxiety regarding the trouble which might have arisen and wanted it to be cleared away.

Ramanujan was shy by temperament and contemplative by nature. He was a man with a great sense of humour.

> *He had a fund of stories, and such was his enjoyment in telling them that in his great days his irrepressible laughter often swallowed the climax of his narrative.* (Neville [15]).

He was very affectionate towards his brothers and his mother, in particular. His wife recollected [29] that he knew astrology and made astrological predictions to some extent and that he knew he would not live beyond 34 years. Sometimes, he is supposed to have made predictions for others also.

Some friends of Ramanujan have remembered [30] that Ramanujan could foresee events in visions; that being an ardent devotee of Lord Narasimha he saw drops of blood in dreams (which was considered as a sign of the Lord's grace) and that

> *after seeing such drops, scrolls containing the most complicated mathematics used to unfold before him, and these he set down on paper on waking only a fraction of what was thus shown to him.*

Ramanujan's maternal grandmother was a staunch devotee of Goddess Namagiri of Namakkal. Ramanujan himself was known to his friends to be a devotee of the Goddess of Namakkal and he used to say that the Goddess appeared in his dreams and inspired him to come forth with new formulae[23].

Prof. K. Ananda Rao was at King's College, when Ramanujan was at Trinity College, and he recalled [32], in 1962, that:

> *In his nature he was simple, entirely free from affectation, with no trace whatever of his being self-conscious of his abilities. He was quite sociable, very polite and considerate to others.*

Ramanujan never forgot that as a first born he had to shoulder the responsibility of taking care of his parents. He was compassionate. Accepting the University's offer of a scholarship, he wrote to Mr. Francis Dewsbury, the Registrar of the University of Madras, in a letter [31] dated 11th January 1919, from a nursing home in Putney:

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[23]This was probably his way of explaining away his incomparable intuition and success, to those who could not comprehend his ability to churn out continuously new results but who persisted in questioning him as to how he arrived at those results!

> *I feel, however, that after my return to India, which I expect to happen as soon as arrangements can be made, the total amount of money to which I shall be entitled will be much more than I shall require. I should hope that, after my expenses in England have been paid, £50 a year will be paid to my parents and that the surplus, after my necessary expenses are met, should be used for some educational purpose, such in particular as the reduction of school-fees for poor boys and orphans and provision of books in schools. No doubt it will be possible to make an arrangement about this after my return.*
>
> *I feel very sorry that, as I have not been well, I have not been able to do so much mathematics during the last two years as before. I hope that I shall soon be able to do more and will certainly do my best to deserve the help that has been given to me.*

Ramanujan concluded a letter [33] to Mr. Narayana Iyer, in November 1915, with the following words of gratitude:

> *I am ever indebted to you and Sir Francis Spring for your zealous interest in my case from the very beginning of acquaintance.*

2. Ramanujan's Mathematics: glimpses

Ramanujan left behind a formidable number of theorems/results in his Notebooks. Prof. Bruce C. Berndt has spent all of his time for over two decades to study each and everyone of the 3254 entries in the Notebooks of Ramanujan. These results and his published work in the *Collected papers* [III], have been a source for further researches to innumerable mathematicians the world-over. In this chapter, an attempt is made to whet the interest of the uninitiated into the profound contributions of Ramanujan.

The breadth, depth and volume of Ramanujan's prolific work and its stupendous consequences on the growth of mathematics, makes this chapter sketchy and jumpy (obviously due to the author's limitations). The purpose of the presentation of some of Ramanujan's results is to give a few glimpses into the nature of the problems thought of by him. This chapter can be bypassed by those who are interested only in the life of Ramanujan, thereby overlooking glimpses at a few of the mathematical gems (theorems) discovered by Ramanujan.

In his first historic letter to Hardy, in 1913, Ramanujan wrote [XI, p.21] :

> *"Dear Sir,*
>
> *I beg to introduce myself to you as a clerk in the Accounts Department of the Port Trust Office at Madras on a salary of only £20 per annum. I am now about 23 years of age. I have had no University education but I have undergone the ordinary school course. After leaving school I have been employing the spare time at my disposal to work at Mathematics. I have not trodden through the conventional regular course which is followed in a University course, but I am striking out a new path for myself. I have made a special investigation of divergent series in general and the results I get are termed by the local mathematicians as 'startling' ...*
>
> *I would request you to go through the enclosed papers. Being poor, if you are convinced that there is anything of value I would like to have my theorems published. I have not given the actual investigations nor the expressions that I get but I have indicated the lines on which I proceed. Being inexperienced I would very highly value any advice you give me. Requesting to be excused for*

the trouble I give you,
I remain, Dear Sir, Yours truly,

S. RAMANUJAN.

This and the second letter of Ramanujan to Hardy contained some 120 formulae from his Notebooks and these are reprinted in the *Collected Papers of Srinivasa Ramanujan* [III] and in *Ramanujan: Letters and Commentary* by Bruce C. Berndt and Robert A. Rankin [XI].

We give below Hardy's selections and comments on some examples from them. Hardy states [35] that:

> *There is always more in one of Ramanujan's formulae than meets the eye, as any one who sets to work to verify those which look the easiest will soon discover. In some the interest lies very deep, in others comparatively near the surface; but there is not one which is not curious or entertaining*[1].

Series formulae

$$1-\frac{3!}{(1!\ \ 2!)^3}\,x^2+\frac{6!}{(2!\ \ 4!)^3}\,x^4-\cdots=$$

$$=\left(1+\frac{x}{(1!)^3}+\frac{x^2}{(2!)^3}+\cdots\right)\left(1-\frac{x}{(1!)^3}+\frac{x^2}{(2!)^3}-\cdots\right) \quad (1)$$

$$1-5\left(\frac{1}{2}\right)^3+9\left(\frac{1.3}{2.4}\right)^3-13\left(\frac{1.3.5}{2.4.6}\right)^3+\cdots=\frac{2}{\pi}. \quad (2)$$

$$1+9\left(\frac{1}{4}\right)^4+17\left(\frac{1.5}{4.8}\right)^4+25\left(\frac{1.5.9}{4.8.12}\right)^4+\cdots=\frac{2^{3/2}}{\pi^{1/2}\{\Gamma(3/4)\}^2}. \quad (3)$$

$$1-5\left(\frac{1}{2}\right)^5+9\left(\frac{1.3}{2.4}\right)^5-13\left(\frac{1.3.5}{2.4.6}\right)^5+\cdots=\frac{2}{\{\Gamma(3/4)\}^4}. \quad (4)$$

> *The series formulae I found much more intriguing, and it becomes obvious that Ramanujan must possess much more general theorems. The second is a formula of Bauer well known in the theory*

[1] These comments and the ones following the equations (numbered up to (12)) are from the introduction to *Collected Papers* of Srinivasa Ramanujan [III].

of Legendre series, but the others are much harder than they look. The theorems required for proving them can all be found now in Bailey's Cambridge Tract on Hypergeometric Functions (1935).

Integrals

$$\int_0^\infty \frac{1+\left(\frac{x}{b+1}\right)^2}{1+\left(\frac{x}{a}\right)^2}\ \frac{1+\left(\frac{x}{b+2}\right)^2}{1+\left(\frac{x}{a+1}\right)^2}\cdots dx = \frac{\pi^{1/2}}{2}\frac{\Gamma(a+1/2)\,\Gamma(b+1)\,\Gamma(b-a+1/2)}{\Gamma(a)\,\Gamma(b+1/2)\,\Gamma(b-a+1)}. \tag{5}$$

$$\int_0^\infty \frac{dx}{(1+x^2)\,(1+r^2x^2)\,(1+r^4x^2)\,\ldots} = \frac{\pi}{2\,(1+r+r^3+r^6+r^{10}+\ldots)}. \tag{6}$$

I thought that, as an expert in definite integrals, I could prove these, and did so, with a good deal more trouble than I had expected. On the whole, the integral formulae seemed least impressive.

If $\alpha\beta = \pi^2$, then

$$\alpha^{-1/4}\left[1+4\alpha\int_0^\infty \frac{xe^{-\alpha x^2}}{e^{2\pi x}-1}dx\right] = \beta^{-1/4}\left[1+4\beta\int_0^\infty \frac{xe^{-\beta x^2}}{e^{2\pi x}-1}dx\right]. \tag{7}$$

Continued fractions

$$\int_0^a e^{-x^2}dx = \frac{\pi^{1/2}}{2} - \frac{e^{-a^2}}{2a} + \frac{1}{a} + \frac{2}{2a} + \frac{3}{a} + \frac{4}{2a} + \cdots \tag{8}$$

The r.h.s. of the last equation contains the continued fraction in its conventional (space-saving) notation, viz.:

$$\frac{a_1}{b_1+\dfrac{a_2}{b_2+\dfrac{a_3}{b_3+\dfrac{a_4}{b_4+\ldots}}}} = \frac{a_1}{b_1} + \frac{a_2}{b_2} + \frac{a_3}{b_3} + \frac{a_4}{b_4} + \cdots.$$

$$4\int_0^\infty \frac{xe^{-x\sqrt{5}}}{\cosh x}dx = \frac{1}{1} + \frac{1^2}{1} + \frac{1^2}{1} + \frac{2^2}{1} + \frac{2^2}{1} + \frac{3^2}{1} + \frac{3^2}{1} + \cdots. \tag{9}$$

> *I had proved things rather like (7) myself, and seemed vaguely familiar with (8). Actually (8) is classical, it is a formula of Laplace first proved properly by Jacobi; and (9) occurs in a paper published by Rogers in 1907.*

The following formulae represent theorems *'on a different level and obviously both difficult and deep'* which even Hardy [36] *'had never seen anything in the least like them before'* and which he has stated *'defeated me completely'*:

$$\text{If } u = \frac{x}{1} + \frac{x^5}{1} + \frac{x^{10}}{1} + \frac{x^{15}}{1} + \cdots \quad \text{and } v = \frac{x^{1/5}}{1} + \frac{x}{1} + \frac{x^2}{1} + \frac{x^3}{1} + \cdots$$

$$\text{then} \qquad v^5 = u\,\frac{1-2u+4u^2-3u^3+u^4}{1+3u+4u^2-2u^3+u^4}. \tag{10}$$

$$\frac{1}{1} + \frac{e^{-2\pi}}{1} + \frac{e^{-4\pi}}{1} + \cdots = \left\{\sqrt{\left(\frac{5+\sqrt{5}}{2}\right)} - \frac{\sqrt{5}+1}{2}\right\} e^{2\pi/5}. \tag{11}$$

$$\frac{1}{1} + \frac{e^{-2\pi\sqrt{5}}}{1} + \frac{e^{-4\pi\sqrt{5}}}{1+\cdots} = \left\{\frac{\sqrt{5}}{1+\sqrt[5]{\left\{5^{3/4}\left(\frac{\sqrt{5}-1}{2}\right)^{5/2}-1\right\}}} - \frac{\sqrt{5}+1}{2}\right\} e^{2\pi/\sqrt{5}} \tag{12}$$

> *A single look at them is enough to show that they could be written down by a mathematician of the highest class. They must be true because, if they were not true, no one would have had the imagination to invent them. Finally (you must remember that I knew nothing whatever about Ramanujan, and I had to think of every possibility), the writer must be completely honest, because great mathematicians are commoner than thieves or humbugs of such incredible skill.*

Hardy considered it *sufficiently marvelous that he should have even dreamt of problems such as these* and *Rogers and Watson found the proofs of the extremely difficult theorems*, in later years.

Elementary Mathematics

As an example of Ramanujan's elementary mathematics which possesses some novelty, we refer to the question 700 posed by him in the Journal of the Indian Mathematical Society ([VII], p.199). Sum the series:

$$(a+b+1)\left(\frac{a}{b}\right)^2+(a+b+3)\left[\frac{a(a+1)}{b(b+1)}\right]^2+(a+b+5)\left[\frac{a(a+1)(a+2)}{b(b+1)(b+2)}\right]^2+\ldots$$

to n terms. The solutions were provided by K.R. Rama Aiyar and K. Appukuttan Erady (Jour.IMS VIII, p.152) and require the definition of

$$u_n=\left[\frac{a(a+1)(a+2)\ldots(a+n-1)}{b(b+1)(b+2)\ldots(b+n-1)}\right]^2 \qquad \text{and} \qquad v_n=(a+b+2n-1)u_n,$$

so that $(b-a-1)v_n=(a+n-1)^2u_{n-1}-(a+n)^2u_n$. Putting $n=2,3,\ldots,n$ in succession in this equation and adding the equations, one finds :

$$v_2+v_3+\ldots+v_n=\frac{(a+1)^2u_1-(a+n)^2u_n}{(b-a-1)}$$

and finally, arrives at the desired sum:

$$v_1+v_2+v_3+\ldots+v_n=\frac{a^2-(a+n)^2u_n}{(b-a-1)}\,, \qquad b\neq a+1.$$

Another example is the solution provided by Ramanujan (J. of IMS IV, p.226) to the Question 289 posed in the Jour. IMS III, p.90. Find the values of:

i) $\sqrt{(1+2\sqrt{(1+3\sqrt{(1+\ldots)}\ \ldots)}}$

ii) $\sqrt{(6+2\sqrt{(7+3\sqrt{(8+\ldots\ldots)}\ldots)}}$

Ramanujan gave the following method to solve these problems:

(i) Notice that $\qquad n(n+2)=n\sqrt{(1+(n+1)(n+3))}$

Let $f(n)=n(n+2)$, then the first of the above identities becomes:

$$f(n)=n\sqrt{(1+f(n+1))}=n\sqrt{(1+(n+1)\sqrt{(1+f(n+2))}}=\ldots$$

$$=n\sqrt{(1+(n+1)\sqrt{(1+(n+2)\sqrt{(1+\ldots)}\ldots)}}$$

Putting $n = 1$, we have $\sqrt{(}1 + 2\sqrt{(}1 + 3\sqrt{(}1 + \ldots)\ldots) = 3$.

(ii) In a similar manner:

$$n(n+3) = n\sqrt{((n+5) + (n+1)(n+4))}$$

Supposing $f(n) = n(n+3)$, we have:

$$f(n) = n\sqrt{((n+5)+f(n+1))} = n\sqrt{((n+5)+(n+1)\sqrt{((n+6)+f(n+2)\ldots)} = \ldots$$

Thus,

$$n(n+3) = n\sqrt{((n+5) + (n+1)\sqrt{((n+6) + (n+2)\sqrt{((n+7) + \ldots))}}.$$

Putting $n = 1$, we have:

$$\sqrt{(}6 + 2\sqrt{(}7 + 3\sqrt{(}8 + \ldots)\ldots) = 4.$$

Ramanujan generalized these two results and made an entry [40a] in his Notebooks as:

$$x + n + a = \sqrt{(}ax + (n+a)^2 + x\sqrt{(}a(x+n) + (n+a)^2 + (x+n)\sqrt{}etc.)\ldots)$$

from which by putting $x = 2, n = 1$ and $a = 0$ and 1 gives the results (i) and (ii), respectively.

This elementary result of Ramanujan was perhaps arrived at by the observations:

$$\begin{aligned} 3 &= \sqrt{9} = \sqrt{1+8} = \sqrt{1+2.4} \\ &= \sqrt{1+2\sqrt{16}} = \sqrt{1+2\sqrt{1+15}} \\ &= \sqrt{1+2\sqrt{1+3.5}} = \sqrt{1+2\sqrt{1+3\sqrt{25}}} \\ &= \sqrt{1+2\sqrt{1+3\sqrt{1+24}}} \\ &= \sqrt{1+2\sqrt{1+3\sqrt{1+4.6}}} \\ &= \sqrt{1+2\sqrt{1+3\sqrt{1+4\sqrt{36}}}} \end{aligned}$$

$$= \sqrt{1+2\sqrt{1+3\sqrt{1+4\sqrt{1+35}}}}$$

$$= \sqrt{(1+2\sqrt{(1+3\sqrt{(1+4\sqrt{(1+5\sqrt{(1+\cdots}}}}}$$

and

$$\begin{aligned} 4 &= \sqrt{16} = \sqrt{6+10} = \sqrt{6+2.5} = \sqrt{6+2\sqrt{25}} \\ &= \sqrt{6+2\sqrt{7+18}} = \sqrt{6+2\sqrt{7+3.6}} = \sqrt{6+2\sqrt{7+3\sqrt{36}}} \\ &= \sqrt{6+2\sqrt{7+3\sqrt{8+28}}} = \sqrt{6+2\sqrt{7+3\sqrt{8+4.7}}} \\ &= \sqrt{(6+2\sqrt{(7+3\sqrt{(8+4\sqrt{(9+5\sqrt{(10+\cdots}}}}} \end{aligned}$$

For another elementary number theoretic problem of solving the equation:

$$x^2 + 7 = 2^n,$$

Ramanujan [III, p.327] conjectured that:

$$x = 1, 3, 5, 11, 181$$

and correspondingly

$$n = 3, 4, 5, 7, 15$$

are the **only** five rational integral solutions for x and n. This conjecture was first proved by T. Nagell [77] only decades later in 1948[2]. In the words of Bruce Berndt [VIII]:

> *Not only are such results fascinating, but for the most part, Ramanujan's methods remain a mystery.*

---

[2]The generalized Diophantine equation $x^2 - D = l^n$, where D and $l > 0$ are specified integers, has been studied by Helmut Hasse [78] who has surveyed the literature, especially for the case $l = 2$, D odd and gave necessary conditions for the existence of the solution. I wish to thank Prof. S. Raghavan for these references, not explicitly given in his article [79].

One of the early papers of Ramanujan was on Highly Composite Numbers, which Hardy called was *of an elementary but highly ingenious character.* Ramanujan had already thought about these numbers since results on these numbers are in his Notebooks. A brief account of these numbers follows, in view of the importance of this work to the career of Ramanujan.

Highly composite numbers

An integer is said to be a *prime* number if it is divisible only by itself and by 1. The first few prime numbers are: 2, 3, 5, 7, 11, 13, 17, 19, 21, ... In other words, if p is a prime, it has only two divisors, a fact which is denoted by $d(p) = 2$. There are infinitely many prime numbers. The number of primes below a given integer and their density has been studied extensively by Legendre and Gauss, among others, and this led to the Prime Number Theorem[3]. A number which is not a prime number is a *composite* number[4].

The Fundamental Theorem of Arithmetic states that every positive integer can be expressed as a product of powers of primes and that this factorization is unique (except for the ordering of the factors). That is, every number can be written as:

$$N = p_1^{a_1} p_2^{a_2} p_3^{a_3} \cdots p_n^{a_n},$$

where $p_1, p_2, \cdots p_n$ are distinct prime numbers, and $a_1, a_2, \cdots a_n$ are non-negative integers. Ramanujan defines

> *a highly composite number*[5] *as a number whose number of divisors exceeds that of all its predecessors.*

He proved that for all highly composite numbers N, except 4 and 36, $a_1 \geq a_2 \geq a_3 \geq \cdots \geq a_n$ and that $a_n = 1$. He denoted the number of divisors of N by $d(N)$ and for the given set of prime numbers $p_1, p_2, \cdots p_n$, $d(N) = (1 + a_1)(1 + a_2)(1 + a_3) \cdots (1 + a_n)$.

We give below a table for the first twenty-five integers (N) with their factorizations, their divisors, the number of divisors $d(N)$ and in the last column

---

[3]The theorem that $\pi(x) \approx \frac{x}{logx}$, $\pi(x)$ being the number of primes not exceeding x, is called the Prime Number Theorem, which was eventually proved, using powerful methods of Complex Analysis, by Hadamard and de la Vallée Poussin, in 1896.

[4]The integer 1 is neither prime nor composite.

[5]Definition: A number N may be said to be a highly composite number, if $d(N') < d(N)$ for all values of N' less than N (Ramanujan [70]).

a remark as to whether the number is a prime, a composite or a highly composite number.

N	Divisors of N	$d(N)$	Remark
$2 = 2^1$	1,2	2	prime, highly composite
$3 = 3^1$	1,3	2	prime
$4 = 2^2$	1,2,4	3	highly composite
$5 = 5^1$	1,5	2	prime
$6 = 2^1 \cdot 3^1$	1,2,3,6	4	highly composite
$7 = 7^1$	1,7	2	prime
$8 = 2^3$	1,2,4,8	4	composite
$9 = 3^2$	1,3,9	3	composite
$10 = 2^1 \cdot 5^1$	1,2,5,10	4	composite
$11 = 11^1$	1,11	2	prime
$12 = 2^2 \cdot 3^1$	1,2,3,4,6,12	6	highly composite
$13 = 13^1$	1, 13	2	prime
$14 = 2^1 \cdot 7^1$	1,2,7,14	4	composite
$15 = 3^1 \cdot 5^1$	1,3,5,15	4	composite
$16 = 2^4$	1,2,4,8,16	5	composite
$17 = 17^1$	1,17	2	prime
$18 = 2^1 \cdot 3^2$	1,2,3,6,9,18	6	composite
$19 = 19^1$	1,19	2	prime
$20 = 2^2 \cdot 5^1$	1,2,4,5,10,20	6	composite
$21 = 3^1 \cdot 7^1$	1,3,7,21	4	composite
$22 = 2^1 \cdot 11^1$	1,2,11,22	4	composite
$23 = 23^1$	1,23	2	prime
$24 = 2^3 \cdot 3^1$	1,2,3,4,6,8,12,24	8	highly composite
$25 = 5^2$	1,5,25	3	composite

Ramanujan starts his paper on Highly Composite Numbers [70] with the following introductory paragraph:

> *The number $d(N)$ of divisors of N varies with extreme irregularity as N tends to infinity, tending itself to infinity or remaining*

> *small according to the form of N. In this paper I prove a large number of results which add a good deal to our knowledge of the behaviour of $d(N)$.*

Hardy has stated that a highly composite number is *as unlike a prime as a number can be.* In his second Notebook and in his classic paper on Highly Composite Numbers [70], Ramanujan has listed the first 102 of these, the first few of which are: 2, 4, 6, 12, 24, 36, 48, 60 and 120. In his table he gives N, $d(N)$ which is also expressed in terms of powers of primes, and $d(d(N)) \equiv dd(N)$. The last entry in his table is:

$$N = 6746328388800 = 2^6 \cdot 3^4 \cdot 5^2 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23$$

$$d(N) = 10080 = 2^5 \cdot 3^2 \cdot 5 \cdot 7, \qquad dd(N) = 72.$$

The problem which was thought of and solved by Ramanujan

> *is a very peculiar one, standing somewhat apart from the main channels of mathematical research. But there can be no question as to the extraordinary insight and ingenuity which he has shown in treating it, nor any doubt that his memoir is one of the most remarkable published in England for many years.*

Ramanujan concludes the introduction to this paper with

> *I prove also that two successive highly composite numbers are asymptotically equivalent, i.e., that the ratio of two consecutive such numbers tends to unity. These are the most striking results. More precise ones will be found in the body of the paper. These results give us a fairly accurate idea of the structure of a highly composite number.*
>
> *I then select from the general aggregate of highly composite numbers a special set which I call "superior highly composite numbers"[6]. I determine completely the general form of all such numbers, and I shew how a combination of the idea of a superior highly composite number with the assumption of the truth of the Riemann hypothesis concerning the roots of the ζ-function leads to*

[6]The first few of these superior highly composite numbers are: 2, 6, 12, 60, 120, 360, 2520, 5040, 554400, 720720, $\cdots$.

even more precise results concerning the maximum order of $d(N)$. These results naturally differ from all which precede in that they depend on the truth of a hitherto unproved hypothesis.

As stated earlier (in Chapter 1) when this paper was submitted to the London Mathematical Society, in 1915, due to the financial difficulties at that time, Ramanujan was requested to reduce the length of the paper. Still, it is his longest paper (62 pages, with 269 equations). It was for this work that Ramanujan was awarded the B.A. Degree by research of the Cambridge University. An interesting anecdote regarding this paper is reported by George Andrews [71]: W.N. Bailey, reputed for his Cambridge University Tract and masterpiece on *Generalized Hypergeometric Functions* [72], was an undergraduate at Cambridge when Ramanujan arrived at Trinity College. Together they had attended a course of lectures by Hardy. In his inaugural lecture on accepting the chair of Professor of Mathematics, Bedford College, he included the following as a tribute to Ramanujan:

... I remember that when Hardy wanted the value of an integral he turned to Ramanujan and asked him for the value, and Ramanujan immediately supplied him with the answer.

Another occasion I remember was being invited by Ramanujan, along with S. Pollard (who afterwards became a lecturer at Trinity) to go to his rooms in Great Court. There he showed us the manuscript of a long paper he had been writing on "Highly Composite Numbers", *which was afterwards published in the Proceedings of the London Mathematical Society. He started at the beginning and quickly turned over the pages as he explained the ideas and the arguments very briefly. Pollard wrestled with the argument and was rewarded by a severe headache. I gave up the struggle earlier and was not troubled by a headache.*

He certainly had a great effect on my own work, and I think that probably, if Ramanujan had not lived, I should not now be the new Professor of Mathematics at Bedford College.

Partitions

It is in the theory of Partitions that Ramanujan and Hardy's collaboration contributed to a remarkable theorem. A partition of an integer n is a division of n into any number of positive integral parts. The number of partitions of n is denoted by $p(n)$. Ramanujan also discovered, for the first time, the congruence properties of $p(n)$.

The first few examples for $p(n)$ are:

$$\begin{aligned}
p(1) &= 1 \quad \text{and} \quad 1 = 1 \\
p(2) &= 2 \quad \text{and} \quad 2 = 2,\ 1+1 \\
p(3) &= 3 \quad \text{and} \quad 3 = 3,\ 2+1,\ 1+1+1 \\
p(4) &= 5 \quad \text{and} \quad 4 = 4,\ 3+1,\ 2+2,\ 2+1+1,\ 1+1+1+1 \\
p(5) &= 7 \quad \text{and} \quad 5 = 5,\ 4+1,\ 3+2,\ 3+1+1,\ 2+2+1, \\
&\qquad\qquad\quad 2+1+1+1,\ 1+1+1+1+1
\end{aligned}$$

It can be seen that $p(n)$ increases rapidly with n, for $p(6) = 11$, $p(7) = 15$, $\cdots p(20) = 627$, $\cdots p(50) = 204,226$ and so on.

In the case of $p(5)$, notice that 5, 4+1 and 3+2 are distinct parts of 5, and 5, 3+1+1, 1+1+1+1+1 are odd parts of 5. The observation that these two parts are equal led Euler, two hundred years ago, to a theorem which is today named after him:

Euler's Theorem: For any postive integer n, the number of partitions of n with distinct parts equals the number of partitions of n with odd parts.

Basic properties of arithmetic can be used to prove this 200-year old theorem. We follow here the lucid exposition of G.E. Andrews in his article (in ref. [IX]) entitled: *Ramanujan: Prodigy and Hero of Mathematics*, wherein the Rogers - Ramanujan identity is formulated as a result in arithmetic. The following example details the step-by-step procedure to transform all partitions of 44 with odd parts into partitions of 44 with distinct parts

e.g. $44 = 7+7+7+7+3+3+3+1+1+1+1+1+1+1$

Step 1. Collect like terms:

$$44 = 4 \times 7 + 3 \times 3 + 7 \times 1,$$

where 4,3,7 are coefficients.
Step 2. Write the coefficients as sums of distinct powers of 2. Thus,

$$44 = (4) \times 7 + (2+1) \times 3 + (4+2+1) \times 1.$$

Step 3. Carry out the indicated multiplications:

$$44 = 28 + 6 + 3 + 4 + 2 + 1.$$

This is now a partition of 44 with distinct parts.

In the next example, all 3 rules are applied to the partitions of 5 with odd parts:

5	=	1×5	=	$(1) \times 5$	=	5
3+1+1	=	$1 \times 3 + 2 \times 1$	=	$(1) \times 3 + (2) \times 1$	=	3+2
1+ 1+ 1+ 1+ 1	=	5×1	=	$(4+1) \times 1$	=	4 +1

Fundamental properties of arithmetic have been used to establish that indeed these 3 rules always provide a proof of Euler's theorem.

A hundred and fifty years after Euler's theorem, Rogers (1894) and Ramanujan (1913) independently found a theorem that is much deeper than Euler's theorem, even though it looks almost the same.

> If *Odd numbers* are characterized by integers with 1,3,5,7 or 9 as last digit, and we call integers with last digit 1,4,6 or 9 as *Strange numbers*, then *the number of partitions of n into strange parts equals the number of partitions of n into distinct parts no two of which are consecutive integers.*

This is the first Rogers-Ramanujan theorem. The following example illustrates this formulation of the Rogers – Ramanujan theorem:

n = 12	has	11 + 1	12
		9+1+1+1	11+1
		6+6	10+2
		6+4+1+1	9+3
		6+1+1+1+1+1+1	8+4
		4+4+4	8+3+1
		4+4+1+1+1+1	7+5
		4+1+1+1+1+1+1+1	7+4+1
		1+1+1+1+1+1+1+1+1+1+1+1	6+4+2
		nine strange parts	nine distinct parts without consecutive integers

The natural question which arises in one's mind is whether the first Rogers – Ramanujan theorem can also be proved in the same way as Euler's theorem, using only properties of arithmetic ?

This problem remained an open problem for 60 years, until A. Garsia and S. Milne found the solution which they published a paper entitled: *A Rogers – Ramanujan bijection*[7].

In the words of George Andrews: *Elegance, depth and surprise, are beautifully intertwined* in this problem thought of by Ramanujan.

Ramanujan's congruences

Very little was known in Ramanujan's times about the arithmetic properties of $p(n)$ - the number of partitions of n. For instance, it was not known whether $p(n)$ is odd or even. Hardy [41] states:

> *Ramanujan was the first and up to now, the only, mathematician to discover any such properties; and his theorems were discovered in the first instance, by observation. MacMahon had calculated a table of $p(n)$ for the first 200 values of n, and Ramanujan observed that the table indicated certain simple congruence properties of $p(n)$.*

[7] Journal of Combinatorial Theory, Ser. A. **31** (1981) 289-339.

Euler, in 1742, proved the identity:

$$\frac{1}{(1-x)(1-x^2)(1-x^3)\cdots} = 1 + \sum_{n=1}^{\infty} p(n)\, x^n \text{ for } \quad |x| < 1.$$

Ramanujan used this identity, the following Euler identity:

$$(1-x)(1-x^2)(1-x^3)\cdots = \sum_{n=-\infty}^{\infty} (-1)^n x^{n(3n+1)/2},$$

and the Jacobi formula:

$$\{(1-x)(1-x^2)(1-x^3)\cdots\}^3 = \sum_{n=-\infty}^{\infty} (-1)^n (2n+1) x^{n(n+1)/2},$$

to prove that $p(5n+4)$ is divisible by 5 for all n. Ramanujan, in 1919, found this and some more congruences concerning $p(n)$. He proved [41] the following:

$$\begin{aligned} p(5n+4) &= 0 \pmod 5, \\ p(7n+5) &= 0 \pmod 7, \\ p(11n+6) &= 0 \pmod{11}. \end{aligned}$$

In other words,

$$\begin{gathered} p(4),\ p(9),\ p(14), \cdots \text{ are divisible by } 5 \\ p(5),\ p(12),\ p(19), \cdots \text{ are divisible by } 7 \\ p(28),\ p(17),\ p(28), \cdots \text{ are divisible by } 11. \end{gathered}$$

For $n = 0$, these congruences imply: $p(4) = 5$, $p(5) = 7$, $p(6) = 11$.

Ramanujan also proved congruences with moduli $5^2, 7^2$ and 11^2:

$$\begin{aligned} p(25n+24) &= 0 \pmod{5^2} \\ p(49n+47) &= 0 \pmod{7^2} \\ p(121n+116) &= 0 \pmod{11^2} \end{aligned}$$

and went a good deal further to put forward a general conjecture:

$$\textit{if} \quad \delta = 5^a 7^b 11^c \quad \textit{and} \quad 24\lambda \equiv 1 \pmod{\delta},$$

$$\textit{then} \quad p(m\delta + \lambda) \quad \equiv \quad 0 \pmod{\delta} \quad \textit{for every } m.$$

This conjecture of Ramanujan led to a considerable amount of work in this area. Hansraj Gupta [42] while extending the table of MacMahon to values of $p(n)$ up to $n = 300$, found that

$$p(243) = 133,978,259,344,888$$

which is not divisible by 7^3. S. Chowla [43] observed that, since

$$24,243 \equiv 1 \pmod{7^3}$$

this contradicts Ramanujan's conjecture. However, Krečmar [44] proved the congruence:

$$p(125n + 99) \equiv 0 \pmod{5^3}.$$

Identities

Two remarkable algebraic identities discovered by Ramanujan are related to the first two congruences. Hardy has stated [45] that:

> *If I had to select one formula from all of Ramanujan's work, I would agree with Major MacMahon in selecting the formula:*
>
> $$p(4) + p(9)x + p(14)x^2 + \cdots = \frac{5\{(1-x^5)(1-x^{10})(1-x^{15})\cdots\}^5}{\{(1-x)(1-x^2)(1-x^3)\cdots\}^6}$$
>
> *where $p(n)$ is the number of partitions of n.*

This formula appears in Ramanujan's paper [46] entitled : *Some properties of $p(n)$, the number of partitions of n*, along with another remarkable identity:

$$p(5) + p(12)x + p(19)x^2 + \cdots = \frac{7\{(1-x^7)(1-x^{14})(1-x^{21})\cdots\}^3}{\{(1-x)(1-x^2)(1-x^3)\cdots\}^4}$$

$$+ \frac{49x\{(1-x^7)(1-x^{14})(1-x^{21})\cdots\}^7}{\{(1-x)(1-x^2)(1-x^3)\cdots\}^8}.$$

These identities make the truth of the divisibility of $p(5n+4)$ and $p(7n+5)$ by 5 and 7, respectively, obvious. Hardy [47] points out that *Ramanujan never published a complete proof* of these two formulae *but proofs have been*

found by Darling and Mordell.

Rogers – Ramanujan identities

In 1917, Hardy [IV, p.91] states that

> *Ramanujan looking through old volumes of the London Mathematical Society, came accidentally across Rogers's papers. I can remember very well his surprise and the admiration which he expressed for Rogers's work.*

The celebrated Rogers – Ramanujan identities were originally posed by Ramanujan, as Q. 584, in the Journal of the Indian Mathematical Society (VI, 1914, p. 199) :

Examine the correctness of the following results:

$$(i)\quad 1+\frac{x}{1-x}+\frac{x^4}{(1-x)(1-x^2)}+\cdots+\frac{x^{m^2}}{(1-x)(1-x^2)\cdots(1-x^m)}+\cdots$$
$$=\frac{1}{(1-x)(1-x^6)(1-x^{11})\cdots}\cdot\frac{1}{(1-x^4)(1-x^9)(1-x^{14})\cdots}$$

here $1,4,9,\cdots$ *on the left are the squares of natural numbers while* $1,6,11,\cdots$ *and* $4,9,14,\cdots$ *on the right are numbers in arithmetical progression, with 5 for common difference.*

$$(ii)\quad 1+\frac{x^2}{1-x}+\frac{x^6}{(1-x)(1-x^2)}+\cdots+\frac{x^{m(m+1)}}{(1-x)(1-x^2)\cdots(1-x^m)}+\cdots$$
$$=\frac{1}{(1-x^2)(1-x^7)(1-x^{12})\cdots}\cdot\frac{1}{(1-x^3)(1-x^8)(1-x^{13})\cdots}$$

here the n^{th} *term of the sequence* $2,6,12,\cdots$ *is* $n(n+1)$ *and* $2,7,12,\cdots$; $3,8,13,\cdots$ *increase by 5.*

> *The formulae have a very curious history. They were found first in 1894 by Rogers, a mathematician of great talent but comparatively little reputation, now remembered mainly from Ramanujan's rediscovery of his work ··· Ramanujan rediscovered the formulae sometime before 1913. He had then no proof (and knew that he had none). ... They are therefore stated without proof in the second volume of MacMahon's Combinatory Analysis. The mystery*

> *was solved, trebly, in 1917. ... A correspondence followed in the course of which Rogers was led to a considerable simplification of his original proof. About the same time I. Schur,.., rediscovered the identities again. Schur published two proofs, one of which is combinatorial and quite unlike any other proof known. There are now seven published proofs, the four referred to already, the two much simpler proofs found later by Rogers and Ramanujan and published in the Papers, and a much later proof by Watson based on quite different ideas. None of these proofs can be called both 'simple' and 'straightforward', since the simplest are essentially verifications and no doubt it would be unreasonable to expect a really easy proof.* (Hardy [48]).

One of the proofs due to Schur was made into a combinatorial proof by Garsia and Milne (1981), who discovered a basic combinatorial fact — their involution principle. These beautiful identities appear in Chapter XVI of Ramanujan's second Note book. This chapter has been thoroughly analyzed and edited by Adiga, Berndt, Bhargava and Watson [49].

The Rogers–Ramanujan identities arise in the study of Euclidian Lie algebras, and the search for a proof of these identities in this setting led to a deeper understanding of some of the representation theory of these algebras. These identities arise naturally in R.J. Baxter's (1980) solution of the hard hexagon model in Statistical Mechanics.

The distribution of a film of liquid Helium on a graphite plate is a physical example of the Hard Hexagon model. Baxter (like Rogers and Schur) found and proved the Rogers–Ramanujan identities without knowing them beforehand. R.J. Baxter's solution of the 2–dimensional Hard Hexagon model is detailed in Ch. 14 of *Exactly Solved Models in Statistical Mechanics* (Acad. Press, 1982).

Baxter's work on the Hard Hexagon Model has led to generalizations, in particular to the work of G.E. Andrews and R.J. Baxter on the *Eight–vertex SOS model and generalized Rogers–Ramanujan–type identities*, published in J. Stat. Phys. **35** (1984) 193. The interested reader is urged to refer to the article of G.E. Andrews on *The Reasonable and Unreasonable Effectiveness*

of Number Theory in Statistical Mechanics (Pennsylvania State University Preprint)[8].

In a one page abstract (Paper 29 in [IV]) Ramanujan announced that he had found a number of algebraic identities for $G(x)$ and $H(x)$:

$$G(x) = \prod_{n=1}^{\infty} \frac{1}{(1-x^{5n-4})(1-x^{5n-1})} \text{ and } H(x) = \prod_{n=1}^{\infty} \frac{1}{(1-x^{5n-3})(1-x^{5n-2})},$$

then

$$H(x)G(x)^{11} - x^2G(x)H(x)^{11} = 1 + 11xG(x)^6H(x)^6$$

and

$$H(x)G(x^{11}) - x^2G(x)H(x^{11}) = 1.$$

Each of these formulae is the simplest of a large class.

Notice that the Rogers-Ramanujan identities, referred to earlier, are:

$$G(x) = \sum_{n=0}^{\infty} \frac{x^{n^2}}{(1-x)(1-x^2)\cdots(1-x^n)}, H(x) = \sum_{n=0}^{\infty} \frac{x^{n(n+1)}}{(1-x)(1-x^2)\cdots(1-x^n)}.$$

Ramanujan's *Lost* Notebook contains 40 such identities, and in particular :

$$H(x)G(x^6) - xG(x)H(x^6) = \frac{P(x)}{P(x^3)},$$

where

$$P(x) = \prod_{n=1}^{\infty}(1-x^{2n-1}).$$

Integrals

The three Quarterly Reports submitted by Ramanujan to the University of Madras, as its first Research Scholar, contain a large number of formulae for

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[8]Also, ref. Andrews [70]. Obviously Andrews' title was inspired by the classic article of Eugene P. Wigner entitled: *The Unreasonable Effectiveness of Mathematics in Natural Sciences*, Communs. on Pure and Applied Maths. **13** (1960) 1.

definite integrals. Hardy [51] refers to a formula of which Ramanujan was fond of:

> *There is one particularly interesting formula, viz.*
>
> $$\int_0^\infty x^{s-1}\{\phi(0) - x\phi(1) + x^2\phi(2) - \cdots\}dx = \frac{\pi\phi(-s)}{sin\ \pi s}$$
>
> *of which he was especially fond and made continual use. This is really an 'interpolation formula', which enables us to say, for example, that, under certain conditions, a function which vanishes for all integral values of its argument must vanish identically, I have never seen this formula stated explicitly by anyone else, though it is closely connected with the work of Mellin and others.*

When $\phi(u) = \frac{\lambda(u)}{\Gamma(1+u)}$ the above formula becomes:

$$\int_0^\infty x^{s-1}\{\lambda(0) - \frac{x}{1!}\lambda(1) + \frac{x^2}{2!}\lambda(2) - \cdots\}dx = \Gamma(s)\lambda(-s).$$

These two integrals are variants of one another and Ramanujan *used them as one of his commonest tools.*

An 'Astonishing' Theorem

Hardy and Ramanujan showed that $p(n)$, the number of unrestricted partitions of n, can be calculated exactly to any value of n. E.C. Titchmarsh in his biographical note on Hardy which appears in the *Collected papers of G.H. Hardy* [XIII] wrote:

> *I must now describe Hardy's work on partitions, the 'circle method', in the analytic theory of numbers and his association with Ramanujan. They wrote five papers together, the most famous being that in Vol.* **17** *of the Proceedings of the London Mathematical Society (1918, 5), a section of which is reproduced on the dust-cover of a Mathematician's Apology.*
>
> *In this it is shown that p(n) the number of unrestricted partitions of n, can not only be represented approximately by an asymptotic*

formula, but that it can be calculated exactly for any value of n. The 'circle method' on which this depends is, no doubt, Hardy's most original creation. It proceeds roughly as follows. The numbers p(n) are the coefficients in the expansion:

$$f(z) = 1 + \sum_{n=1}^{\infty} p(n)z^n = \frac{1}{(1-z)(1-z^2)(1-z^3)\cdots}$$

$$\textit{so that} \qquad p(n) = \frac{1}{2\pi i}\int_{\Gamma} \frac{f(z)}{z^{n+1}}dz$$

where Γ is a path enclosing the origin and lying entirely inside the unit circle and is taken to be a concentric circle of radius just < 1. The unit circle is a line of essential singularities of $f(z)$, but certain points are found to have a particularly strong influence on the integral, and it is from these that a dominant term is ultimately derived. In the Dirichlet series method for proving e.g. the prime-number theorem, the dominant term is easily identified, and almost the whole difficulty lies in showing that it is dominant.

A whole apparatus, involving the Farey dissection of the circle and the linear transformations of the elliptic modular functions, is needed to produce the result. It is all the more astonishing that the analysis should have been carried out to the point at which the exact value of p(n) could be obtained.

Similar methods were applied later by Hardy and Littlewood to many problems, particularly to the Waring problem of the expression of a number as a sum of given powers, and to problems involving primes. One such problem which had long defied analysis was Goldbach's theorem, or hypothesis, that any even number can be expressed as the sum of two primes. The success of the circle method in the study of p(n) suggests a similar approach to Goldbach's theorem. ... But the original Goldbach hypothesis still remains unproved.

In an 'astonishing theorem' (Littlewood [51]) Hardy and Ramanujan have shown that the number $p(n)$ of the partitions of n is the integer nearest to

$$\frac{1}{\pi\sqrt{2}}\sum_{q=1}^{\nu}\sqrt{q}A_q(n)\Psi_q(n),$$

where

$$A_q(n) = \sum \omega_{p,q} e^{-2np\pi i/q},$$

the sum being over all p prime to q and less than it, $\omega_{p,q}$ is a certain 24 q-th root of unity, ν is of the order of $\sqrt{n}$, and

$$\Psi_q(n) = \frac{d}{dn}\left[exp\{\frac{\pi}{q}\sqrt{\frac{2}{3}}(n-1/24)\}\right].$$

Taking $\nu = 5$, for $n = 200$, they predicted the value:

$$p(200) = 3,972,999,029,388$$

which was computed by Major MacMahon to be correct. Littlewood [51], while reviewing Ramanujan's 'Collected Papers' has given an exciting account of the collaboration between the two men *of quite unlike gifts, in which each contributed the best, most characteristic, and most fortunate work that was in him.* Later on, D.H. Lehmer showed that the Hardy-Ramanujan series was divergent and Rademacher refined the circle method [52] using what are called Ford Circles, which simplified considerably the estimate and led him to a convergent series for $p(n)$.

In this context, it is worthwhile to record here the comments of the Fields Medalist Atle Selberg on $p(n)$, [76]:

> *The paper by Hardy and Ramanujan contained surely a result that was very remarkable in itself; since $p(n)$ is an integer, it allows exact computation on it. But it was not an exact formula. It was a formula with an error tending to zero as n grows and therefore $p(n)$ being an integer, one could find the exact value.*

If one looks at Ramanujan's first letter to Hardy, there is a statement there which has some relation to his later work on the partition function, namely about the coefficient of the reciprocal of

a certain theta series (a power series with square exponents and alternating signs as coefficients). It gives the leading term in what he claims as an approximate expression for the coefficient. If one looks at that expression, one sees that this is the exact analogue of the leading term in the Rademacher formula for p(n) which shows that Ramanujan, in whatever way he had obtained this, had been led to the correct term of that expression.

In the work on partition function, studying the paper it seems clear to me that it must have been, in a way, Hardy who did not fully trust Ramanujan's insight and intuition, when he chose the other form of the terms in their expression, for a purely technical reason, which one analyses as not very relevant. I think that if Hardy had trusted Ramanujan more, they should have inevitably ended with the Rademacher series. There is little doubt about that.

Elliptic and modular functions

Euler (1748) gave the following integral representation for the hypergeometric function:

$$F(a,b;c;z) = \frac{\Gamma(c)}{\Gamma(b)\Gamma(c-b)} \int_0^1 t^{b-1}(1-t)^{c-b-1}(1-tz)^{-a}dt,$$

where $Re\ c > Re\ b > 0$ and $|arg(1-z)| < \pi$. It is straightforward to show that, for $|k| < 1$,

$$K(k) = \frac{1}{2}\pi F(\frac{1}{2},\frac{1}{2};1;k^2) = \int_0^1 \frac{dt}{\sqrt{(1-t^2)(1-k^2t^2)}}$$

and

$$E(k) = \frac{1}{2}\pi F(-\frac{1}{2},\frac{1}{2};1;k^2) = \int_0^1 \frac{\sqrt{(1-k^2t^2)}}{\sqrt{(1-t^2)}}dt.$$

These fundamental integrals are called complete Legendre *elliptic integrals* of the first and second kind. The complementary integrals E' and K' are integrals with the complementary variable: $k' = \sqrt{(1-k^2)}$. Note that for complementary variables: $f'(k) = f(k')$, and the variable k is called the

modulus and the variable k' is called the *complementary modulus* and the prime does not denote differentiation, in this context. In terms of k', the complementary elliptic integrals E' and K' are:

$$K'(k) = K(k') \qquad \text{and} \qquad E'(k) = E(k').$$

The reason why these are called *elliptic* integrals is because the perimeter A of an ellipse, with semi-major and semi-minor axes being a and b, is given by $A = 4aE'(\frac{b}{a})$. Elliptic functions can be expressed as inverse functions of elliptic integrals. Jacobi (1829) is the originator of the fascinating theory of theta functions, an important class of functions closely related to the elliptic functions. It is the considered opinion of some [XIV] that Ramanujan had not seen any standard book on elliptic functions, before he went to England, since he used his own notation and his work does not reveal his awareness of the fundamental double periodicity property[9] of the elliptic functions.

Hardy in *Ramanujan* ([IV], Chapter XII) says that it is in Ramanujan's work on elliptic and modular functions

> *that both the profundity and the limitations of Ramanujan's knowledge stand out most sharply...*
> *Ramanujan never professed to have made any major advance in the theory of elliptic functions, and it seems that he must have learnt the fundamentals of the theory, so far as he was interested in them, from books. There is a sharp contrast between his attitude here and his attitude about the theory of primes, where he certainly regarded all his results his own. He never writes as if he had 'invented' theta-functions or modular equations, though he sets out a whole theory of them in a language of his own. Cayley's and Greenhill's books were in the Madras University Library, and he could have found a good deal about them there. Indeed Littlewood says that "Ramanujan somehow acquired an effectively complete knowledge of the formal side of the theory of elliptic functions",*

[9] A function of a complex variable $f(z)$ is called a *doubly periodic* function of z, if $f(z) = f(z + 2\omega_1) = f(z + 2\omega_2)$, where ω_1, ω_2 are two complex numbers for which the ratio ω_1/ω_2 is not a real number and $2\omega_1$, $2\omega_2$ are the periods of the function. An elliptic function is a doubly periodic function which is meromorphic in the finite part of the complex plane. (Analytic functions $f(z)$ that have only well separated poles as singularities are called meromorphic).

> *and that his ignorance of complex function theory and Cauchy's theorem may seem difficult to reconcile with this; and adds that "a sufficient, and I think necessary, explanation would be that Greenhill's very odd and individual 'Elliptic Functions' was his text-book". In Greenhill's book the complex variable and double periodicity are not mentioned until p. 254, and the double periodicity is deduced somehow from properties of Cartesian ovals. In fact Greenhill knew very little more "function theory" than Ramanujan.*

Hardy also states ([IV], p.10) that

> *And there is some of his work, mostly in the theory of elliptic functions, about which some mystery still remains; it is not possible, after all the work of Watson and Mordell, to draw the line between what he may have picked up somehow and what he must have found for himself.*

Chapters 16 to 21 of the second Notebook of Ramanujan, dealt with q-series and θ-functions; the fundamental properties of elliptic functions; Jacobi's elliptic functions; modular equations of degrees 3, 5, 7 and associated θ-function identities; modular equations of higher and composite degrees and Eisenstein series, respectively. Bruce Berndt has studied thoroughly the 834 results contained in these six chapters in Part III of his work [VIII]. G.N. Watson considers that these results *show Ramanujan at his best.*

'Mock'-theta functions

By theta-functions, Ramanujan apparently means [53] sums, products and quotients of series of the form:

$$\sum_{n=-\infty}^{\infty} (-)^{\varepsilon n} q^{an^2+bn} \qquad \text{where } \varepsilon = 0 \text{ or } 1.$$

Examples of theta-functions are:

$$\sum_{-\infty}^{\infty} (-1)^n q^{6n^2+4n} = 1 - q^2 - q^{10} + q^{16} + q^{32} - \cdots.$$

and

$$\sum_{\infty}^{\infty} q^{6n^2+4n} = 1 + q^2 + q^{10} + q^{16} + q^{32} + \cdots$$

The transformed Eulerian form of the theta-functions are:

$$1 + \frac{q}{(1-q)^2} + \frac{q^4}{(1-q)^2(1-q^{2)^2}} + \frac{q^9}{(1-q)^2(1-q^2)^2(1-q)^3)^2} + \cdots$$
$$= \frac{1}{\sum_{n=-\infty}^{\infty} (-1)^n q^{n(3n-1)/2}}$$

which is an identity of Euler, and

$$1 + \frac{q}{1-q} + \frac{q^4}{(1-q)(1-q^2)} + \frac{q^9}{(1-q)(1-q^2)(1-q^3)} + \cdots$$
$$= \frac{\sum_{n=-\infty}^{\infty} (-1)^n q^{n(5n-1)/2}}{\sum_{n=-\infty}^{\infty} (-1)^n q^{n(3n-1)/2}}$$

which is the first Rogers-Ramanujan identity.

In his last letter to Hardy, in Jan. 1920, Ramanujan wrote [54]:

> *I am extremely sorry for not writing to you a single letter up to now ... I discovered very interesting functions recently which I call* Mock *theta-functions. Unlike the* False *theta-functions (studied partially by Prof. Rogers in his interesting paper) they enter into mathematics as beautifully as the ordinary theta-functions. I am sending you with this letter some examples...*
>
> *I have proved that if*
>
> $$f(q) = 1 + \frac{q}{(1+q)^2} + \frac{q^4}{(1+q)^2(1+q^2)^2} + \cdots$$
>
> *then*
>
> $$f(q) + (1-q)(1-q^3)(1-q^5)\cdots(1-2q+2q^4-2q^9+\cdots) = O(1)$$

at all the points $q = -1, q^3 = -1, q^5 = -1, q^7 = -1, \ldots,$ *and at the same time*

$$f(q) - (1-q)(1-q^3)(1-q^5)\cdots(1-2q+2q^4-\cdots) = O(1)$$

at all the points $q^2 = -1, q^4 = -1, q^6 = -1, \ldots$. *Also obviously* $f(q) = O(1)$ *at all the points* $q = 1, q^3 = 1, q^5 = 1, \ldots$. *And so* $f(q)$ *is a Mock* θ *function. When* $q = -e^{-t}$ *and* $t \to 0$

$$f(q) + \sqrt{\frac{\pi}{t}} exp\left(\frac{\pi^2}{24t} - \frac{t}{24}\right) \to 4.$$

The coefficient of q^n *in* $f(q)$ *is*

$$(-1)^{n-1}\frac{exp\left(\pi\sqrt{\frac{n}{6}-\frac{1}{144}}\right)}{2\sqrt{n-\frac{1}{24}}} + O\left(\frac{exp\left(\frac{\pi}{2}\sqrt{\frac{n}{6}-\frac{1}{144}}\right)}{\sqrt{n-\frac{1}{24}}}\right).$$

It is inconceivable that a single θ *function could be found to cut out the singularities of* $f(q)$.

Mock theta-functions:

$$\phi(q) = 1 + \frac{q}{1+q^2} + \frac{q^4}{(1+q^2)(1+q^4)} + \cdots$$

$$\Psi(q) = \frac{q}{1-q} + \frac{q^4}{(1-q)(1-q^3)} + \frac{q^9}{(1-q)(1-q^3)(1-q^5)} + \cdots$$

$$\chi(q) = 1 + \frac{q}{1-q+q^2} + \frac{q^4}{(1-q+q^2)(1-q^2+q^4)} + \cdots$$

These are related to $f(q)$ *as shown below.*

$$2\phi(-q) - f(q) = f(q) + 4\Psi(-q) = \frac{1-2q+2q^4-2q^9+\cdots}{(1+q)(1+q^2)(1+q^3)\cdots}$$

$$4\chi(q) - f(q) = \frac{(1-2q^3+2q^{12}-\cdots)^2}{(1-q)(1-q^2)(1-q^3)\cdots}$$

These are of the 3rd order.

Mock theta-functions (of 5th order):

$$f(q) = 1 + \frac{q}{1+q} + \frac{q^4}{(1+q)(1+q^2)} + \frac{q^9}{(1+q)(1-q^2)(1-q^3)} + \cdots$$

$$\phi(q) = 1 + q(1+q) + q^4(1+q)(1+q^3) + q^9(1+q)(1+q^3)(1+q^5) + \cdots$$

$$\Psi(q) = q + q^3(1+q) + q^6(1+q)(1+q^2) + q^{10}(1+q)(1+q^2)(1+q^3) + \cdots$$

$$\chi(q) = 1 + \frac{q}{1-q^2} + \frac{q^2}{(1-q^3)(1-q^4)} + \frac{q^3}{(1-q^4)(1-q^5)(1-q^6)} + \cdots$$

$$= 1 + \left\{ \frac{q}{1-q} + \frac{q^3}{(1-q^2)(1-q^3)} + \frac{q^5}{(1-q^3)(1-q^4)(1-q^5)} + \cdots \right\}$$

$$F(q) = 1 + \frac{q^2}{1-q} + \frac{q^8}{(1-q)(1-q^3)} + \cdots$$

$$\phi(-q) + \chi(q) = 2F(q).$$

$$f(-q) + 2F(q^2) - 2 = \phi(-q^2) + \Psi(-q)$$

$$= 2\phi(-q^2) - f(q) = \frac{1 - 2q + 2q^4 - 2q^9 + \cdots}{(1-q)(1-q^4)(1-q^6)(1-q^9)\cdots}$$

$$\Psi(q) - F(q^2) + 1 = q\frac{1 + q^2 + q^6 + q^{12} + \cdots}{(1-q^8)(1-q^{12})(1-q^{28})\cdots}$$

... ...

Mock theta-functions (of 7th order):

$$(i) \quad 1 + \frac{q}{1-q^2} + \frac{q^4}{(1-q^3)(1-q^4)} + \frac{q^9}{(1-q^4)(1-q^5)(1-q^6)} + \cdots$$

$$(ii) \quad \frac{q}{1-q} + \frac{q^4}{(1-q^2)(1-q^3)} + \frac{q^9}{(1-q^3)(1-q^4)(1-q^5)} + \cdots$$

$$(iii) \quad \frac{1}{1-q} + \frac{q^2}{(1-q^2)(1-q^3)} + \frac{q^6}{(1-q^3)(1-q^4)(1-q^5)} + \cdots$$

These are not related to each other.

By *False* theta functions are meant simply the theta series with the *wrong* signs like :

$$\sum_{n=0}^{\infty}(-1)^n q^{6n^2+4n}(1+q^{4n+2}) = 1+q^2-q^{10}-q^{16}+q^{32}+\cdots$$

Hardy has commented that

> *a mock theta function is a function defined by a q-series convergent when $|q|<1$ for which we can calculate asymptotic formulae, when q tends to a rational point, $e^{2\pi ir/s}$ of the unit circle, of the same degree of precision as those furnished for the ordinary theta functions by the theory of linear transformations.*

The classical one-dimensional heat equation and the Korteweg-de Vries equation have been solved in terms of theta functions. In recent times, theta functions have been found to be of great use in the theory of Solitons – a field of great interest in Modern Physics. Freeman J. Dyson, a renowned theoretical physicist, on the occasion of the Ramanujan birth centenary stated [XVI]:

> *My dream is that I will live to see the day when young physicists, struggling to bring the predictions of superstring theory into correspondence with the facts of nature, will be led to enlarge their analytic machinery to include not only theta-functions, but also mock theta-functions.*

Continued fractions

It is Ramanujan's results on continued fractions, contained in his first historic letter to Mr. Hardy, in 1913, which made the latter state that the formulae *defeated* him completely (p.42 of this book). Ramanujan's major work on continued fractions revolves around the generalization of what is called the *Ramanujan continued fraction*:

$$\frac{1}{1}+\frac{x}{1}+\frac{x^2}{1}+\frac{x^3}{1}+\cdots, \qquad |x|<1. \tag{1.1.1}$$

Prof. K.G. Ramanathan, the renowned mathematician of the Tata Institute of Fundamental Research, Bombay, in his *Homi J. Bhabha medal lecture – 1984* entitled: *Ramanujan's continued fraction* states[55]:

This continued fraction was, however, first discovered by L.J. Rogers in his beautiful work[10] *(p. 328) on expansions of infinite products. It was rediscovered around 1911 by Ramanujan who recognized its importance and applied to it the methods of elliptic and modular functions. Indeed, in his, now famous, first letter to Hardy in 1913, he stated a number of beautiful results about (1.1.1) which led Hardy to believe, and rightly too, that these could have been written down only by a mathematician of the highest class. In the 'Lost' note book, he gave a number of generalizations of (1.1.1) and their evaluations at points in an imaginary quadratic field.*

Atle Selberg[11] *in 1936, working independently, rediscovered almost all of Ramanujan's generalizations of (1.1.1) and proved them in an elementary manner. His methods seem to be of wider applicability. Issai Schur*[12]*, independently of both Rogers and Ramanujan, rediscovered (1.1.1) and the Rogers-Ramanujan identities. Some of Schur's results on (1.1.1) are also to be found in the 'Lost' note book.*

Ramanujan has done pioneering work on continued fractions. He has asserted that

$$1+\frac{aq}{1}+\frac{aq^2}{1}+\frac{aq^3}{1}+\cdots+\frac{aq^n}{1}=\frac{A_n}{B_n}$$

$$A_n = 1 + aq\frac{1-q^n}{1-q} + a^2q^4\frac{(1-q^{n-1})(1-q^{n-2})}{(1-q)(1-q^2)} + \cdots$$

$$B_n = 1 + aq^2\frac{1-q^{n-1}}{1-q} + a^2q^6\frac{(1-q^{n-2})(1-q^{n-3})}{(1-q)(1-q^2)} + \cdots$$

for $|q| < 1$. Though this is the only identity which appeared in his published papers, his notebooks, particularly chapter XII of the second note book, ex-

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[10] L.J. Rogers, *Second memoir on the expansion of certain infinite products*, Proc. London Math. Soc. **25** (1894) 318–43.

[11] A. Selberg, *Über einige arithmetische Identitäten*, Avhand. Utigitt. Norske Vid. Akad. Oslo. (1936) No. 8, 23 pages.

[12] Issai Schur, *Ein Beitrag zur additiven Zahlentheorie und zur Theorie der Kettenbrüche*, Sitz Preuss. Akad. Wiss. Physikalisch-Math. Klasse (1917) 302–21 (=Ges. Abhandlungen Bd 2 Springer (1973) 117–136).

tensivesly edited in recent times by Bruce C. Berndt ([XI], part II) and the 'Lost' notebook contain a large number of very beautiful results on continued fractions. In a series of papers [56], Prof. Andrews has undertaken the task of grouping and proving the fomulae contained in the 'Lost' notebook. Having completed the task of editing the note books of Ramanujan, Prof. Bruce C. Berndt[13] is teaming up with Prof. George E. Andrews, the discoverer of the 'Lost' note book, to edit it.

Ramanujan's sum

The sum

$$c_q(n) = \sum_{p(q)} e^{-2np\pi i/q}$$

which is a sum over p less than and prime to q (or any complete system of residues prime to q) is called the Ramanujan sum. Ramanujan summed a large number of series of the form:

$$\sum_q a_q c_q(n).$$

Arithmetic functions

The second letter of Ramanujan to Hardy , dated Feb. 27, 1913, contains the following identity involving hypergeometric functions of unit argument:

$$\text{if} \quad \frac{F(\frac{1}{4},\frac{3}{4},1,1-\alpha)}{F(\frac{1}{4},\frac{3}{4},1,\alpha)} = 5\frac{F(\frac{1}{4},\frac{3}{4},1,1-\beta)}{F(\frac{1}{4},\frac{3}{4},1,\beta)}$$

then

$$(\alpha\beta)^{1/2} = [(1-\alpha)(1-\beta)]^{1/2} + [\alpha\beta(1-\alpha)(1-\beta)]^{1/6}[(\alpha\beta)^{1/6} + ((1-\alpha)(1-\beta))^{1/6}].$$

It is after seeing this identity that Prof. Littlewood remarked that Ramanujan *is at least a Jacobi*, when he was asked by Hardy to comment on the formulae contained in the first two letters of Ramanujan to Hardy (in 1913).

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[13]Private communication, Oct. 1997.

Ramanujan developed in a simple and purely algebraic way the basic properties of the classical elliptic and other allied functions in his paper: *On certain elliptic functions.* For instance he gave an *a priori* simple proof of the following identity:

$$\{\frac{1}{4}cot(\frac{\theta}{2}) + \sum_{n=1}^{\infty} x^n \frac{sin(n\theta)}{(1-x^n)}\}^2 = \{\frac{1}{4}cot(\frac{\theta}{2})\}^2 + \sum_{n=1}^{\infty} x^n \frac{cos(n\theta)}{(1-x^n)^2}$$

$$+\frac{1}{2}\sum_{n=1}^{\infty} nx^n \frac{(1-cos(n\theta))}{(1-x^n)},$$

which forms the foundation of his theory. For, from this identity he derives many interesting results. Venkatachaliengar [XIV] has remarked that this identity is contained in Jordan's *Cours de Analyze II* in a somewhat different form. Ramanujan himself remarks in his paper that the elementary proofs of these formulae contained in his paper *seem to be of some interest in itself.* Ramanujan has derived the basic properties of elliptic functions also in a purely algebraic way.

Ramanujan defined the symmetric function:

$$\phi_{r,s} = \sum_m \sum_n m^r n^s q^{mn} = \phi_{s,r}$$

for natural numbers r, s, such that $rs \neq 0$ and $(r+s)$ odd. Bernoulli numbers B_r occur in

$$\phi_{2r-1,0} = \sum_n n^{2r-1} \frac{q^n}{1-q^n} - (-1)^r \frac{B_r}{4r} = \phi_{0,2r-1}.$$

Ramanujan proved that $\phi_{r,s}$ is a polynomial over the field of rational numbers in P, Q, R, which are the following functions introduced by him :

$$\begin{aligned} P &= 1 - 24\sum_{n=1}^{\infty} \frac{nq^n}{1-q^n} \\ Q &= 1 + 240\sum_{n=1}^{\infty} \frac{n^3q^n}{1-q^n} \\ R &= 1 - 504\sum_{n=1}^{\infty} \frac{n^5q^n}{1-q^n}, \quad \text{for} \quad |q| < 1. \end{aligned}$$

Ramanujan used these functions to not only derive the classical identity named after Jacobi :

$$Q^3 - R^2 = 1728q \prod_{n=1}^{\infty}(1-q^n)^{24},$$

in the simplest algebraic way, but also discovered many new identities. (ref. K. Venkatachaliengar [XIV]).

Ramanujan's τ - function

The Ramanujan τ -function is defined by

$$\begin{aligned} g(x) &= x\{(1-x)(1-x^2)(1-x^3)\cdots\}^{24} \\ &= x\prod_{n=1}^{\infty}(1-x^n)^{24} = \sum_{n=1}^{\infty}\tau(n)x^n. \end{aligned}$$

Ramanujan tabulated $\tau(n)$ up to n =30 and showed that

$$\tau(p) \equiv 0 \pmod{p} \quad \text{for} \quad p = 2, 3, 5, 7, 23$$

and for these primes

$$\tau(pn) \equiv 0 \pmod{p}, \text{ for every } \quad n.$$

Ramanujan was the first to observe interesting congruence properties of this τ function, such as:

$$\tau(n) = \sigma_r(n) \pmod{691}, \quad \sigma_r(n) = \sum d^r$$

where the sum is over all d dividing n. The prime number 691 which occurs here (and in the Bernoulli number $B_6 = -691/2730$) is called the Ramanujan Prime Number.

The Riemann ζ-function is defined as: $\zeta(s) = \sum_{n=1}^{\infty}\frac{1}{n^s}$, for $s \geq 1$, and the Dirichlet series is defined as $F(s) = \sum_{n=1}^{\infty}\frac{f(n)}{n^s}$. Ramanujan discovered the factorization of the Dirichlet series for his τ-function :

$$\sum_{n=1}^{\infty}\frac{\tau(n)}{n^s} = \prod_{all\ primes}\{1-\tau(p)p_s + p^{11-2s}\}^{-1}.$$

Ramanujan also conjectured :

$$|\tau(p)| \leq 2p^{\frac{11}{2}} \quad \text{for every prime} \quad p.$$

Deligne proved this famous conjecture of Ramanujan, in 1974, using advanced methods of Algebraic Geometry and this work won for him the Fields medal! Another Fields Medalist, Prof. Atle Selberg, in 1988, at an extemporaneous talk at the Tata Institute of Fundamental Research observed [76]:

> *that a felicitous but unproved conjecture may be of much more consequence for mathematics than the proof of many a respectable theorem.*
>
> *Ramanujan's recognition of the multiplicative properties of the coefficients of modular forms that we now refer to as cusp forms and his conjectures formulated in this connection, and their later generalization, have come to play a more central role in the mathematics of today, serving as a kind of focus for the attention of quite a large group of the best mathematicians of our time. Other discoveries like the mock-theta functions are only in very early stages of being understood and no one can yet assess their real importance. So the final verdict is certainly not in, and it may not be in for a long time, but the estimates of Ramanujan's stature in mathematics certainly have been growing over the years. There is no doubt about that.*

Commenting on the quality of the theorems in the 'Lost' Notebook, Richard Askey [60] says:

> *Try to imagine the quality of Ramanujan's mind, one which drove him to work unceasingly while deathly ill, and one great enough to grow deeper while his body became weaker. I stand in awe of his accomplishments; understanding is beyond me. We would admire any mathematician whose life's work was half of what Ramanujan found in the last year of his life while he was dying.*

Bruce Berndt [58] has drawn attention to the following assessments of Ramanujan and his work :

• *In notes left by B.M. Wilson, he tells us how George Polya was captivated by Ramanujan's formulas. One day in 1951 while Polya was visiting Oxford, he borrowed from Hardy his copy of Ramanujan's notebooks. A couple of days later, Polya returned them in almost a state of panic explaining that however long he kept them, he would have to keep attempting to verify the formulae therein and never again would have time to establish another original result of his own.*

Neville began a broadcast in Hindustani, in 1941, with the declaration :

• *Srinivasa Ramanujan was a mathematician so great that his name transcends jealousies, the one superlatively great mathematician whom India has produced in the last thousand years.*

• *Paul Erdös has passed on to us Hardy's personal ratings of mathematicians: Suppose that we rate mathematicians on the basis of pure talent on a scale from 0 to 100, Hardy gave himself a score of 25, Littlewood 30, Hilbert 80, and Ramanujan 100.*

I conclude this chapter, in which I have essentially strung together the statements of several great men of the past and the present, with the observation of Dr. S. Chandrasekhar the Nobel Laureate and renowned Astrophysicist:

As long as people do mathematics, the work of Ramanujan will continue to be appreciated.

3. Ramanujan's Notebooks

The history of the notebooks, in brief, is the following: Ramanujan had noted down the results of his researches, without proofs, (as in *A Synopsis of Elementary Results, a book on pure Mathematics,* by G.S. Carr), in three notebooks, between the years 1903 - 1914, before he left for England. These were the notebooks which he showed to his benefactors to convince them about his abilities as a mathematician. The results in these notebooks were organized by him. The first notebook has 16 chapters in 134 pages. The second is a revised, enlarged version of the first, containing 21 chapters in 252 pages. The third notebook contains 33 pages of unorganized material (included at the end of volume 2 of the facsimile edition [VII]). Ramanujan took these notebooks with him to Cambridge. But, in one of his letters to a friend, he wrote that he had no time to look into them and most probably he did not put them to use during his five year stay abroad.

The Preface to the *Collected Papers of Srinivasa Ramanujan* ([III], p.ix) contains the following appraisal of the work in the notebooks:

> *There is still a large mass of unpublished material. None of the contents of Ramanujan's notebooks has been printed, unless incorporated in later papers, except that one chapter, on generalized hypergeometric series, was analyzed by Hardy* [37] *in the Proceedings of the Cambridge Philosophical Society. This chapter is sufficient to show that, while the notebooks are naturally unequal in quality, they contain much which should certainly be published. It would be a very formidable task to work through them systematically, select particular passages, and edit these with adequate comment, and it is impossible to print the notebooks as they stand without further monetary assistance. The singular quality of Ramanujan's work and the romance which surrounds his career, encourage us to hope that this volume may enjoy sufficient success to make possible the publication of another.*

Hardy tried to persuade the University of Madras to undertake such a task and in turn, Prof. G.N. Watson was requested by the Madras University, in 1931, to edit the notebooks in a suitable form for publication. This was a formidable task since the notebooks contained about 4000 theorems.

Prof. Watson gave a lecture [39] on Ramanujan's notebooks at the London Mathematical Society in 1931, and published several papers between 1928–36, motivated by Ramanujan's work. Watson's Presidential address to the London Mathematical Society, in 1935, was on *mock-theta functions*, discovered by Ramanujan during the period when he was terminally ill (March 1919 – April 1920) after his return to India. Watson undertook the task of editing the notebooks with Prof. B.M. Wilson and

> *they focussed their attention on the second notebook. Chapters 2–13 were to be edited by Wilson, and Watson was to examine chapters 14–21. Unfortunately, Wilson passed away prematurely in 1935 at the age of 38.* (Berndt [40]).

This possibly put an end to the joint effort of Watson and Wilson to edit the notebooks.

In 1957, with monetary assistance from Sir Dadabai Naoroji Trust, at the instance of Prof. Homi J. Bhabha and Prof. K. Chandrasekaran of the Tata Institute of Fundamental Research, the Tata Institute published a facsimile edition of the notebooks of Ramanujan in two volumes. The reproduction is thoroughly faithful to the original. About a thousand copies were made then and in the centenary year of Ramanujan's birth, 1987, Springer - Narosa reprinted the same.

The formidable task of truly editing the notebooks –*viz.* to either prove each of the results or provide references to the literature where the proofs may be found – which was started by G. N. Watson and B. M. Wilson but never completed, was taken up in right earnest by Prof. Bruce C. Berndt, who has been publishing a series of papers, since 1981, by himself or in collaboration with others, including Indian Mathematicians Padmini T. Joshi, C. Adiga and S. Bhargava. He has also utilized the earlier work of G.N. Watson and B.M. Wilson on the notebooks with the Trinity College. This dedicated work of Berndt, published by Springer-Verlag [VIII], is now available in five parts, the first of which appeared in 1985 and the fifth, in 1997.

At the beginning of his work, in Part I, Berndt [34] points out:

> *The notebooks were originally intended primarily for Ramanujan's own personal use and not for publication ... Some of Ramanujan's incorrect 'theorems' in number theory found in his letters to*

Hardy have been well publicized. Thus, perhaps, some think that Ramanujan was prone to making errors. However, such thinking is erroneous. The notebooks contain scattered errors. Especially if one takes into account the roughly hewn nature of the material and his frequently formal arguments, Ramanujan's accuracy is amazing.

Resurgence of interest

After Ramanujan returned to India, though he was seriously ill and bedridden for most of the time, he was working continuously. His life-long habit had been to work on a slate and transfer the final results to the notebook or sheets of paper. His brother-in-law has stated [59] that *a vast quantity of papers containing his notes was handed over to the University after his death.* Three days after Ramanujan died, his brother S. Lakshminarasimhan, in a letter to Prof. Hardy, dated 29th April 1920, informing him about his demise, wrote :

> *All his Mss that were in his trunk were handed over on the day of his death to Mr. Ramachandra Row's son in law, since the former is at Nilgiris. Not only those but also the journals, magazines - all he possessed except the books which he had, were taken away by them.*

In a letter to Prof. Hardy, dated 3 Dec. 1920, Mr. R. Ramachandra Row, wrote:

> *Ramanujan's M.S.S whatever they are are with me and will be handed over intact to the University of Madras who I understand is already in correspondence with you regarding the methods of publication.*

These papers reached Prof. Hardy, who should have passed them on to Prof. G.N. Watson and hence to their final destination – viz. the estate of Prof. Watson, in the underground cellar of Trinity College – after he wrote several papers inspired Ramanujan's results, until their discovery in 1976 by George E. Andrews.

Prof. Watson died in Feb. 1965. A decade later, in the spring of 1976, when Prof. George Andrews [53] of the Pennsylvania State University was going through the estate of Watson, he discovered a box of papers.

> *The most interesting item in this box was a manuscript of more*

> *than one hundred pages in Ramanujan's distinctive handwriting which contains over six hundred mathematical formulae listed one after the other without proof. It is my contention that this manuscript, or notebook, was written during the last year of Ramanujan's life after his return to India from England. My evidence for this assertion is all indirect; in the words of Stephen Leacock, 'It is what we call circumstantial evidence — the same that people are hanged for'* .

These are now referred to as the contents of the *Lost* notebook of Ramanujan. Andrews assessed ([101], p.xii):

> *While it is impossible to categorize the various formulas completely, a rough approximation of its contents is the following:*
>
> *q-series and related topics including mock θ-functions: 60%*
> *Modular equations and relation, singular moduli: 30%*
> *Integrals, Dirichlet series, congruences, asymptotics, misc.: 10%*
>
> *I give only this rough break down because of the chaotic nature of this manuscript. On many pages there are fragments of numerical computations and infinite series running off in all directions. It may be possible eventually to make sense out of some of these pages; if so, the above percentage breakdown may change. ...*
> *In any event it is clear that q-series investigations make up the bulk of the work in in the Lost Notebook. Indeed I count about 380 formulae that I consider to belong more to q-series than to modular relations or other topics.*

Sixty-seven years after the death of Ramanujan, due to the discovery of the 'lost' notebook by Prof. George Andrews, in 1976, and the editing of the three Notebooks of Ramanujan by Prof. Bruce Berndt, there has been a resurgence of interest in the work of Ramanujan. These notebooks of Ramanujan have formed the basis for numerous papers by many mathematicians, who gave proofs of the theorems and conjectures of Ramanujan obtained by him through his intuition and sheer brilliance. This is a singular and unparalleled phenomenon in the annals of mathematics. In the centenary year of his birth, 1987, several conferences were held in many countries to focus attention on his work and the recent developments. Mystery still surrounds *some of his*

work, mostly in the theory of elliptic functions, wherein

> *it is not possible, after all the work of Watson and Mordell, to draw the line between what he may have picked up somehow and what he must have found for himself.* ([IV], p.10).

Hardy [60] regretted that he could easily have but did not find out all the details from Ramanujan himself since he

> *was quite able and willing to give a straight answer to a straight question, and not in the least disposed to make a mystery of his achievements.*

However, Hardy [60] argues that Ramanujan

> *was not particularly interested in his own history or psychology; he was a mathematician anxious to get on with the job. And after all I too was a mathematician, and a mathematician meeting Ramanujan had more interesting things to think about than historical research. It seemed ridiculous to worry him about how he had found this or that known theorem, when he was showing me half a dozen new ones almost every day.*

A Page from the second Notebook

To provide a feeling for Ramanujan's notebooks, we reproduce here a page [61] from his second Notebook, Chapter XVIII. There is no particular reason for choosing this page except that it contains an entry which has become a 'folklore' which illustrates the attachment Ramanujan had to numbers prompting Littlewood to state that to Ramanujan every number is a *personal friend.*

There are 2 parts shown in this page. The first part is concerning the geometrical construction of a square whose area is equal to that of a given circle. This is a rare piece of Ramanujan's work in geometry, since he did very little work in geometry and perhaps did not have much interest in it. Several attempts on this problem have been made from early times by (ancient Greek and Hindu) mathematicians and it has been shown that it is impossible

Pa i. To Construct a Square equal to a given circle.

Let O be the Centre and PR any diameter.

Bisect OP at H and trisect OR at T. Draw TQ perp to OP.

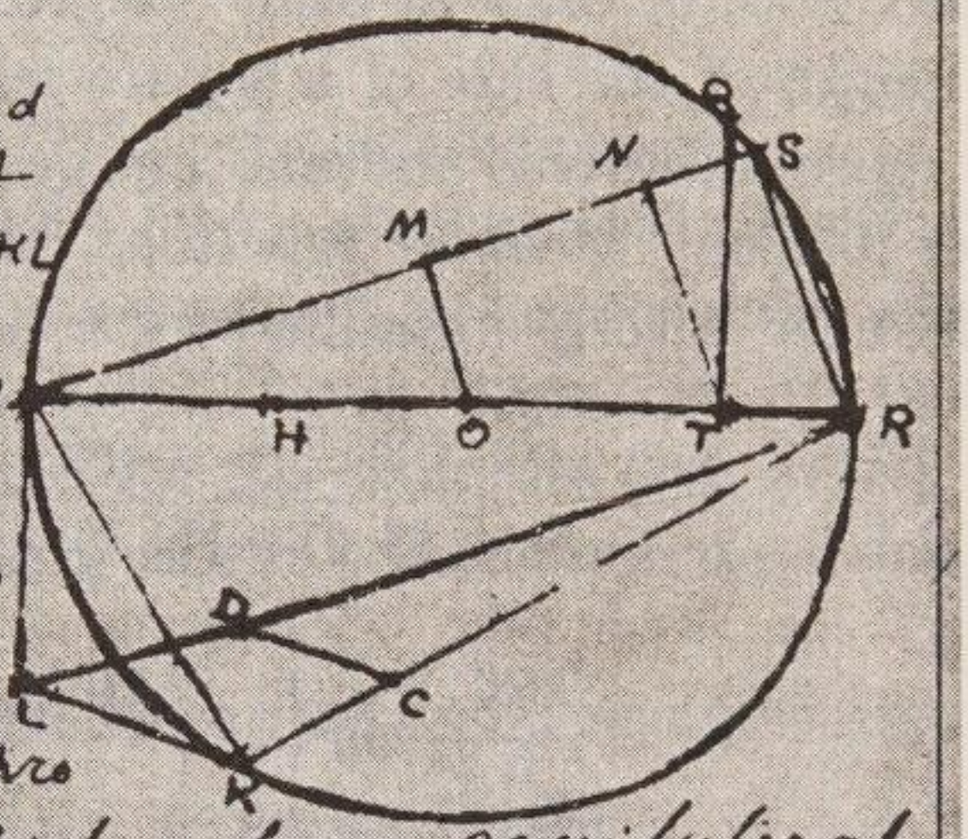
Draw RS = TQ Join PS

Draw OM & TN ll^{l} to RS.

Draw PK = PM; & PL = MN and perp to OP. Join KL, RK & KL

Cut off RC = RH. Draw CD ll^{l} to KL

Then $RD^2 = \odot PQR$.



N.B. RD is $\frac{1}{100}$th of an inch greater than the true length if the given ⊙ is 14 Sq. miles in area.

Cor. 1. One of the two mean proportionals between a side of an equilateral triangle inscribed in the ⊙ and the length PS is one less than $\frac{1}{30000}$th part of it than the true length.

Cor. 2. The app. length got by assuming $\pi = \sqrt[4]{97\frac{1}{2}} - \frac{1}{11}$ is $\frac{1}{100}$th of an inch less than the true length if the ⊙ is a million Square miles in area

ii. $\{6n^2 + (3n^2 - n)\}^3 + \{6n^2 - (3n^2 - n)\}^3 = \{6n^2(3n^2 + 1)\}^2$

iii. $\{m^7 - 3m^4(1+b) + m(3.1+b^2 - 1)\}^3$

$+ \{3m^6 - 3m^4(1+2b) + (1+3b+3b^2)\}^3$

$+ \{m^6 - (1+3b+3b^2)\}^3 = \{m^7 - 3m^4 b + m(3b^2 - 1)\}^3$

ex. $(11\frac{1}{2})^3 + (\frac{1}{2})^3 = 29^2$; $(3 - \frac{1}{105})^3 + (\frac{1}{105})^3 = (5\frac{6}{35})^2$.

$(3\frac{1}{7})^3 - (\frac{1}{7})^3 = (5\frac{1}{7})^2$; $(3\frac{1}{102})^3 - (\frac{1}{102})^3 = (5\frac{23}{102})^2$.

$3^3 + 4^3 + 5^3 = 6^3$; $1^3 + 12^3 = 9^3 + 10^3$; $1^3 + 75^3 = (76\frac{1}{2})^3 + (43\frac{1}{2})^3$

$3^3 + 509^3 + 376^3 = 1188^3$; $18^3 + 19^3 + 21^3 = 28^3$

$7^3 + 14^3 + 17^3 = 20^3$; $19^3 + 60^3 + 69^3 = 82^3$; $15^3 + 82^3 + 89^3 = 108^3$; $3^3 + 36^3 + 37^3 = 46^3$; $1^3 + 135^3 + 138^3 = 173^3$.

Fig. caption: The last three lines of p. 224 and the p. 225 from the second notebook of Ramanujan, chapter XVIII. ([61]).

to *square the circle*, in the Euclidian sense[1]. Hardy has commented that he found in Ramanujan's collection of books in Cambridge some books by 'quacks' on this theme.

Ramanujan published a paper [62] entitled, "*Squaring the circle*" in the Jour. of the I.M.S. in 1913, in which he gives a geometrical construction for finding the length of the side of a square whose area equals that of the circle. He also reproduced this and another geometrical construction for π in his later paper [63] on "*Modular equations and approximations to* π". In this paper, he deduced several formulae[2] for π like:

$$\frac{63}{25}\frac{17+15\sqrt{5}}{7+15\sqrt{5}} = 3.14159265380...$$

$$\pi = \frac{24}{\sqrt{142}} log\left\{\sqrt{\left(\frac{10+11\sqrt{2}}{4}\right)} + \sqrt{\left(\frac{10+7\sqrt{2}}{4}\right)}\right\}$$

$$\pi = 12\sqrt{190}\ log\{(2\sqrt{2}+\sqrt{10})(3+\sqrt{10})\}$$

formulae correct to 9, 16 and 18 decimal places; and

$$\frac{1}{\pi} = 2\sqrt{2}\left\{\frac{1103}{99^2} + \frac{27493}{99^6}\cdot\frac{1}{2}\cdot\frac{1.3}{4^2} + \frac{53883}{99^10}\cdot\frac{1.3}{2.4}\cdot\frac{1.3.5.7}{4\cdot 8^2} + \cdots\right\}$$

$$= 2\sqrt{2}\sum_{n=0}^{\infty}\frac{(1/4)_n(1/2)_n(3/4)_n}{(1)_n(1)_n(1)_n}(1103+26390n)(\frac{1}{99})^{4n+2},$$

which he asserted would be "*rapidly convergent*", for the first term itself gives the sum to 8 decimal places. In 1986, Borwein and Borwein [64], two computer scientists used a version of Ramanujan's formula to calculate π to

---

[1]It is the ancient Greeks who set-up the circle-squaring problem with the following two conditions [83]: the solution should use only a straightedge and compass, so that the proofs can be reduced to Euclid's theorems and the solution must use only a finite number of steps. If, in fact, $\pi r^2 = a^2$, where r is the radius of the circle and a is the length of the side of a square, then it follows that $\pi = (a/r)^2$, or that π is rational. In fact, π has been shown to be not only an *irrational number* but also, proved by Ferdinand von Lindermann, in 1882, to be a *transcendental* number. Furthermore, the phrase 'squaring the circle' in common usage suggests a project doomed to failure!

[2]Note that Ramanujan has used the equality sign for his approximations to π even though he states explicitly that the formulae are correct only to a certain number of decimal places.

17 million places and found that the formula converges on the exact value with far greater efficiency than any previous method[3]. This success proved that Ramanujan's insight was correct.

The next entries on this page in Ch. XVIII of his second note book are solutions of Diophantine equations:

$$X^3 + Y^3 = U^2$$

and $$X^3 + Y^3 + Z^3 = U^3 \quad \text{(Euler's equation)}.$$

which are followed by numerical examples for these equations – 6 solutions for the first and 12 solutions for the Euler's equation and one which looks like but is not a solution of the Euler's equation. Characteristically, Ramanujan does not give the values of the parameters for the examples (ref. [VIII], part III, p.199 for the examples with the values of the parameters appended).

Ramanujan's solution of Euler's equation in rational numbers given here is:

$$\begin{aligned} X &= m^7 - 3m^4(1+p) + m(2+6p+3p^2), \\ Y &= 2m^6 - 3m^3(1+2p) + 1 + 3p + 3p^2, \\ Z &= m^6 - 1 - 3p - 3p^2, \\ U &= m^7 - 3m^4 p + m(3p^2 - 1), \end{aligned}$$

where m and p denote arbitrary numbers. He has also given another solution of this equation elsewhere (see Question 441 which follows) but neither of these two parameter solutions is the general solution[4] which contains three parameters, prompting Hardy [65] to comment that

> *Diophantine equations should have suited him, but he did comparatively little with them, and what he did was not his best.*

The second of the 12 examples given for the Euler equation on this page:

$$1^3 + 12^3 = 9^3 + 10^3 \quad (= 1729)$$

---

[3]David and Gregory Chudnovsky are two mathematicians who hold several world records for calculating the highest number of digits of π: first up to 450 million digits, then 1 billion and then 2 billion digits. The current record is over 51 billion digits (cf. For a brief history of π, [83]).

[4]The last of the examples $1^3 + 6^3 + 8^3 = 9^3$ does not belong to this class of solutions, since no values of m and p yield this solution to the Euler's equation. So, its presence reveals that perhaps, Ramanujan was aware that his solution was not the most general for Euler's equation.

corresponds to $(m = 2, p = 3)$. This has been made famous by the following anecdote associated with it, due to Hardy [66]:

> *He could remember the idiosyncrasies of numbers in an almost uncanny way. It was Littlewood who said that every positive integer was one of Ramanujan's personal friends. I remember going to see him once when he was lying ill in Putney. I had ridden in taxi-cab number 1729, and remarked that the number seemed to me rather a dull one, and that I hoped that it was not an unfortunate omen. 'No', he replied, 'it is a very interesting number, it is the smallest number expressible as a sum of two cubes in two different ways'. I asked him, naturally, whether he could tell me the solution of the corresponding problem for fourth powers; and he replied, after a moment's thought, that he knew no obvious example, and supposed that the first such number must be very large.*

The simplest known solution of:

$$x^4 + y^4 = z^4 + t^4$$

is Euler's: $59^4 + 158^4 = 133^4 + 134^4 = 635318657$.
Euler gave a solution involving two parameters, but no 'general' solution is known.

The anecdote reveals Ramanujan's remarkable feeling for numbers and his sharp memory which made him recall one entry out of several thousands he had made in his notebook just like that and the fact that he had not recorded in his notebook the observation he made about 1729 which came from him only when Prof. Hardy made an innocuous statement regarding the taxi number.

Though the number 1729 itself finds no explicit mention in the notebooks, the Euler's equation and/or its solution: $1^3 + 12^3 = 9^3 + 10^3$ finds mention[5] in three Questions to the Journal of the I.M.S. ([III], p.331) and in two places in his notebooks ([VII], Vol. 2, p.266 and 387). These are indicated below:

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[5]I am thankful to Mr. P.K. Srinivasan for drawing my attention to these and his publications [81].

• **Question 441** (V, 39)[6]:
Shew that

$$(6a^2-4ab+4b^2)^3=(3a^2+5ab-5b^2)^3+(4a^2-4ab+6b^2)^3+(5a^2-5ab-3b^2)^3,$$

and find other quadratic expressions satisfying similar relations.
[*Solution* by S. Narayanan, VI, 226.]

• **Question 661** (VII, 119):
Solve in integers $x^3+y^3+z^3=u^6$,
and deduce the following:

$$\begin{aligned} 6^3-5^3-3^3 &= 2^6, & 8^3+6^3+1^3 &= 3^6,\\ 12^3-10^3+1^3 &= 3^6, & 46^3-37^3-3^3 &= 6^6,\\ 174^3+133^3-45^3 &= 14^6, & 1188^3-509^3-3^3 &= 34^6. \end{aligned}$$

[*Solutions by* N.B. Mitra, XIII, 15 – 17. Additional solution and remarks by N.B. Mitra, XIV, 73 – 77.]

• **Question 681** (VII, 160):
Solve in integers $x^3+y^3+z^3=1$,
and deduce the following:

$$\begin{aligned} 6^3+8^3 &= 9^3-1, & 9^3+10^3 &= 12^3+1,\\ 135^3+138^3 &= 172^3-1, & 791^3+812^3 &= 1010^3-1,\\ 11161^3+11468^3 &= 14258^3+1, & 65601^3+67402^3 &= 83802^3+1. \end{aligned}$$

[*Partial solution by* N.B. Mitra, XIII, 17. See also N.B. Mitra, XIV, 73–77 (76–77).]

• If p, q, r are quantities so taken that $p+3a^2=q+3ab=r+3b^2=(a+b)^2$ and m and n are any two quantities, then

$$n(mp+nq)^3+m(mq+nr)^3=m(np+mq)^3+n(nq+mr)^3.$$

A particular case of the above theorem is:

$$(3a^2+5ab-5b^2)^3+(4a^2-4ab+6b^2)^3+(5a^2-5ab-3b^2)^3=(6a^2-4ab+4b^2)^3.$$

([VIII], Vol.2, p.266)

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[6] J. of the IMS, vol. V, p.39.

- If $\alpha^2 + \alpha\beta + \beta^2 = 3\lambda\gamma^2$, then

$$(\alpha + \lambda^2\gamma)^3 + (\lambda\beta + \gamma)^3 = (\lambda\alpha + \gamma)^3 + (\beta + \lambda^2\gamma)^3.$$

([VIII], Vol.2, p.387)

Note that in the above 4-parameter solution for the Euler's equation, it has been observed (cf. [95] (c), p.43) that for $\alpha = 3, \beta = 0, \lambda = 3$ and $\gamma = 1$, we get $12^3 + 1^3 = 10^3 + 9^3 = 1729$. The fact that Ramanujan could come out spontaneously with the statement that 1729 in the smallest number which could be expressed as the sum of two cubes in two different ways, something which he had not jotted down in his notebooks, proves that every integer is a 'personal friend' of Ramanujan as remarked by Prof. Littlewood.

Hardy's statement that two thirds of Srinivasan Ramanujan's work in India, (contained in the notebooks), consisted of rediscoveries, is refuted by Prof. Bruce C. Berndt, who states after his extensive studies that Hardy's estimate is too high. Berndt's main goal was: "*To prove each of Ramanujan's theorems*" and for known results refer to literature where proofs may be found. Tables of the contents of "Ramanujan's Notebooks", Parts I to V, edited by Prof. Berndt are given here in the following pages. The titles of the chapters gives the reader an idea about the nature and range of problems and the number of results Ramanujan had recorded in each chapter.

In the Introduction to Part V, Berndt states:

> *This volume, however, should not be regarded as the closing chapter on Ramanujan's notebooks. Instead, it is just the first milestone on our journey to understanding Ramanujan's ideas. ... It is our fervent wish that these volumes will serve as springboards for further investigations by mathematicians intrigued by Ramanujan's remarkable ideas. ... Ramanujan remarked that several of his series arose from alternative theories of elliptic functions ... The first of the three alternative theories is the most interesting and the most important, and we feel that a large body of work remains to be discovered here.*

Ramanujan's Note Books, Part I, Bruce C. Berndt ([VIII], 1985):

Ch.	Subject	# of Results
1.	Magic Squares	43
2.	Sums Related to the Harmonic series or the Inverse Tangent Function	68
3.	Combinatorial Analysis and Series Inversions	86
4.	Iterates of the Exponential Function and an Ingenius Formal Technique	50
5.	Eulerian Polynomials and Numbers, Bernoulli Numbers and the Riemann Zeta-Function	94
6.	Ramanujan's Theory of Divergent series	61
7.	Sums of Powers, Bernoulli Numbers and the Γ Function	110
8.	Analogues of the Gamma Function	108
9.	Infinite Series Identities, Transformations, and Evaluations	139

Ramanujan's Quarterly Reports

Ramanujan's Note Books, Part II, Bruce C. Berndt ([VIII], 1989):

Ch.	Subject	# of Results
10.	Hypergeometric Series I	116
11.	Hypergeometric Series II	103
12.	Continued Fractions	113
13.	Integrals and Asymptotic Expansions	92
14.	Infinite Series	87
15.	Asymptotic Expansions Modular Forms	94

Ramanujan's work on hypergeometric series, contained in chapters X and XI of his second notebook was edited by Hardy, as stated earlier. In chapter 4 of this book, we take a closer look at some of the beautiful work of Ramanujan on hypergeometric series.

Ramanujan's Notebooks, Part III, Bruce C. Berndt ([VIII], 1991):

Ch.	Subject	# of Results
16.	q-Series and Theta Functions	134
17.	Fundamental Properties of Elliptic Functions	162
18.	The Jacobian Elliptic Functions	135
19.	Modular Equations of Degrees 3, 5 and 7 and Associated Theta Function Identities	185
20.	Modular Equations of Higher & Composite Degrees	173
21.	Eisenstein series	45

The 100 Pages of Unorganized material after Chapter 21 of Srinivasa Ramanujan's second Notebook and the 33 Pages of his third Notebook were organized into chapters 22 to 31 by Berndt.

Ramanujan's Notebooks, Part IV, Bruce C. Berndt ([VIII], 1994):

Ch.	Subject	# of Results
22	Elementary results	47
23.	Number Theory	108
24.	Theory of Prime numbers	24
25.	Theta Function and Modular Equations	86
26.	Inversion Formulas for Lemniscate & other functions.	10
27.	Q series	9
28.	Integrals	63
29.	Special Functions	39
30.	Partial Fraction Expansions	15
31.	Elementary and miscellaneous analysis	36
	16 Chapters of first Notebook	54

In the concluding part V, Berndt examines the unorganized pages in all the three notebooks of Ramanujan on continued fractions, alternative theories of elliptic functions, class invariants and singular moduli, explicit values of theta-functions, modular equations, infinite series, approximations and asymptotic expansions, etc. Berndt points out that *very few claims in this volume pertain to Ramanujan's published papers and problems.*

Ramanujan's Notebooks, Part V, Bruce C. Berndt ([VIII], 1997)[7]:

Ch.	Subject	# of Results
32.	Continued Fractions	73
33.	Ramanujan's Theories of Elliptic Functions to Alternative Bases	62
34.	Class Invariants and Singular Moduli	196
35.	Values of Theta-Functions	24
36.	Modular Equations and Theta-Function Identities in Notebook 1	87
37.	Infinite Series	53
38.	Approximations and Asymptotic Expansions	46
39.	Miscellaneous Results in the First Notebook	24

Many theorems communicated by Ramanujan to Hardy in his Jan.16, Feb. 27, 1913 letters are found in Chapters.22 – 31. In all, $759+605+834+491+565 = 3254$ theorems have been studied in the parts I–V by Prof. Berndt. Hardy had estimated that the notebooks of Ramanujan contained approximately 3000 – 4000 statements of theorems and besides editing one chapter of Ramanujan's second Notebook, in 1923, he tried to get them edited. His intentions have at long last been fulfilled by the tireless efforts of the indefatigable Bruce C. Berndt. For the motivated student of Mathematics, this

[7]The author thanks Prof. Berndt for readily communicating the Preface and the Contents pages of Part V, by e-mail, on request.

five part publication will provide a plethora of results for further studies. As stated elsewhere [XV], it is no exaggeration to say that as long as people do mathematics, the work of Ramanujan and the stupendous effort of Prof. Bruce C. Berndt in editing the Ramanujan notebooks will be appreciated.

R.P. Agarwal's work on the notebooks

Prof. Ratan P. Agarwal is the founder of a school of ordinary and basic hypergeometric series in India. In recent times, he has consolidated a part of his work in a publication entitled: *Resonance of Ramanujan's Mathematics* [82]. This two volume work concerns some of the results of Srinivasa Ramanujan (1887 - 1920), the renowned Indian Mathematician, recorded in his notebooks, which continue to be a perennial source of inspiration for generations of Mathematicians.

Volume I has five chapters. Chapter 1 concerns certain summation and asymptotic formulae for generalized (ordinary and basic) hypergeometric series. Chapter 2 is on Rogers-Ramanujan identities. Chapters 3 - 5 deal with certain definite Integrals of interest to Ramanujan.

Prof. Agarwal opines, like Hardy, that *a major part of Ramanujan's systematic work on ordinary hypergeometric series has been in the nature of a rediscovery of the work already done by previous authors* – the ${}_7F_6(1)$ summation formula of Dougall (1907) and particular cases of this sum, such as, Pfaff (1797) - Saalschütz (1890) theorem for the ${}_3F_2(1)$ and the Gauss and Kummer summation theorems for the ${}_2F_1(1)$ and the ${}_2F_1(-1)$, respectively, and Dixon's summation theorem for a well-poised ${}_3F_2(1)$. These and Ramanujan's entries in his notebooks concerning the deep results on the asymptotic relations for a finite number of terms of a variety of ordinary hypergeometric series, which center around the behaviour of the logarithmic solutions of the hypergeometric differential equation are presented in Chapter 1.

Chapter 2 deals with the derivations of the celebrated Rogers–Ramanujan identities[8], their combinatorial interpretations, their generalizations and their

[8]Refer chapter 2 of this work for a discussion about these famous identities.

applications in Statistical Mechanics for the solution of the two-dimensional hard-hexagonal model (by R.J. Baxter). Several proofs of these identities and the interesting mathematical developments have been described very well in this cogently written chapter.

The next three chapters of Volume I deal with some of the integrals studied by Ramanujan. Prof. Agarwal presents definite integrals associated with Fourier transforms in the second Notebook and self-reciprocal functions given in his 'lost' Notebook in Chapter 3. He has shown how some of Ramanujan's results on integrals in his Notebook can be established with the help of other entries of his

> *that every entry given in the notebooks by him has a purpose, although sometimes a particular entry may look out of context at the first instance. ... We have advocated elsewhere also that it may be interesting many a time to try to prove 'unproven' entry of Ramanujan with the help of some other entry given by him earlier or later on.*

Chapter 4 is focused on Ramanujan's Master Theorem for the evaluation of definite integrals, viz.:

$$\int_0^\infty x^{n-1}F(x)dx = \Gamma(n)\phi(-n), \qquad \text{where } F(x) = \sum_{k=0}^{\infty} \frac{\phi(k)(-x)^k}{k!},$$

in some neighbourhood of $x = 0$ and for n not necessarily a positive integer. The three Quarterly Reports submitted by Ramanujan to the Madras University, as a requirement stipulated by the University for awarding him the *First Research Scholarship in Mathematics*, contain various applications of this theorem to the *evaluation of a huge variety of integrals* and expansion formulae. The appropriate conditions of convergence for its validity have been provided by Hardy in his lectures contained in *Ramanujan* [IV]. The q-extension of the Master Theorem by Jackson (1951) and its extension to two variables by R.P. Agarwal (1974) are presented. The fundamental gamma and beta functions of mathematical analysis have been ingeniously extended by Ramanujan and these are related to the 'incomplete' gamma and beta functions (in which the upper limit of the integrals is finite and not ∞). The q-extensions of these works by several mathematicians including

Thomae (1879), F.H. Jackson (1904), R. Askey (1978), G.E. Andrews and R. Askey (1981), A. Verma and V.K. Jain (1992) are also included.

Chapter 5 is devoted to the study of integrals of the type:

$$\int_{-\infty}^{\infty} \frac{e^{at^2+bt}}{e^{ct}+d} dt$$

for particular values of a, b, c, d, evaluated by Ramanujan. Mordell (1933) classified these according to the values of the parameters and evaluated them in terms of Jacobi's theta and other related functions, using the method of complex contour integration. Professor Agarwal presents the general method of evaluation of different standard forms of this integral and relates them to those studied by Ramanujan, who used transform calculus to evaluate his integrals. Integrals associated with Riemann's zeta functions and elliptic modular relations are not discussed, as pointed out by the author himself. The chapter concludes with the mention of the example of a definite integral with fractional derivatives in the integrand stated in the third Quarterly Report. Ramanujan's formal deduction is based on his Master theorem and it agrees with the classical Liouville's definition of a fractional derivative of a function.

In Volume II, Prof. Agarwal presents a critical appraisal of Ramanujan's extensive and intriguing work on elliptic functions. Results pertaining to theta functions, partial theta functions, 'mock' theta functions (discovered by Ramanujan during the last year of his life on his return to India), as well as Lambert series and their relationship with elliptic functions, 'mock' theta functions and allied functions. While theta functions and partial theta functions are covered in chapter 1 of this volume and Lambert series and related functions are dealt with in chapter 5, the major part of this volume is concerned with the 'mock' theta function results of Ramanujan, presented in three extensive chapters (2, 3 and 4), contained in the 'Lost' Notebook, which was made available to the world in a facsimile edition, by Narosa, for the first time in the centenary year of Ramanujan's birth 1987 (though it was discovered in the estate of G.N. Watson by Prof. George E. Andrews of the Pennsylvania State University, in Spring 1976).

After giving the definitions of the four Jacobi theta functions and the 'false' theta functions (viz. theta functions with the 'wrong' signs for the terms) defined by L.J. Rogers, Prof. Agarwal introduces the Ramanujan notation for the theta function in terms of his symmetric function:

$$f(a,b) = \sum_{k=-\infty}^{\infty} (ab)^{k(k-1)/2}(a^k + b^k), \quad |ab| < 1.$$

In terms of this symmetric function, Ramanujan defines:

$$f(qe^{2iz}, qe^{-2iz}) = \theta_3(z,q), \text{ where } q = e^{i\pi\tau} \text{ and } |q| < 1.$$

Several general properties of the function $f(a,b)$ are then presented. Jacobi's triple product identity and other such identities due to D. Hickerson (1988) and G.E. Andrews and D. Hickerson (1991) are derived explicitly, due to their importance to simplify calculations in later chapters on 'mock' theta functions. A number of theta function expansions in the 'lost' notebook pertain to 'partial' theta functions. Some of these are stated and G.E. Andrews (1981) derived a general basic hypergeometric identity and showed the results of Ramanujan as special cases of it.

Ramanujan defined four third order mock theta functions and to this set three more were added by Watson. Watson used the transformations of basic hypergeometric series for obtaining new definitions for all the seven functions. Agarwal himself has contributed significantly to the theory of 'mock' theta functions by showing that most of the general identities of Andrews (referred above) belong to a very general class of basic hypergeometric transformations. His results are presented and those of Andrews deduced from them. Furthermore, Agarwal defines the third order mock theta functions in terms of a ${}_2\Phi_1$ basic hypergeometric series and deduces many of the mock theta function properties from the well-known properties of the ${}_2\Phi_1$. Ramanujan gave ten mock theta functions of order five, in two groups of five each, and three functions of order seven. He asserted that the members of each of the two groups of mock theta functions of order five are related amongst those belonging to the same group only, while the three mock theta functions of order seven are not related to each other. The works of Andrews, Agarwal and M. Gupta on the mock theta functions of order five are presented in detail. A. Gupta and Agarwal showed that mock theta functions of order five and

seven are defined through basic hypergeometric series of type ${}_3\Phi_2$ and ${}_4\Phi_3$. They have also made attempts to develop the theory of the above types of basic hypergeometric series to obtain new transformations and definitions for certain mock theta functions of orders five and seven. It is pointed out that the absence of a general transformation theory for the ${}_3\Phi_2(a, b, c; e, f; z)$ series [except when the argument z is q (the base parameter itself) or ef/abc], is the main reason why there exists no general transformation theory for mock theta functions of order five.

Observations of Agarwal on the relationship between the 'mock' theta functions and the basic hypergeometric series lead him to define:

> *the order of a mock theta function as* $(2r + 1)$ *if it is expressible in terms of a* ${}_{r+1}\Phi_r$ *series, on a single base* q^k, $k \leq r + 1$. *There may be in the definition of the mock theta function an additive term with* ${}_{r+1}\Phi_r$ *consisting of* θ*-products, which do not affect the order.*

While this is an intersting observation by itself, it is simplistic, to say the least. For, mystery still surrounds the incomplete work of Ramanujan in this area contributing to the belief that Ramanujan was working on a general theory of mock theta functions, whose preliminary results are the ones in the 'lost' notebook of his and that he was snatched away by fate before he could reveal his grand scheme!

This two volume work of Prof. Agarwal is a deep study of some of the work of Ramanujan on ordinary and basic hypergeometric series, on definite integrals and theta, partial theta and mock theta functions. It is very original in parts and has a minimum overlap with the comprehensive five part work on the notebooks of Ramanujan by Bruce C. Berndt[9]. This work presents, in as coherent a way as is possible, the work of Agarwal and his school of students and coworkers. It is a valuable supplement for the research scholar and for mathematicians and presents the author's understanding of the 'how' and 'why' of what Ramanujan did.

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[9]The author's reference to these Parts I to V as the Berndt Notebooks is unfortunate, especially in view of the significance of the Ramanujan notebooks and the 'Lost' notebook, which are the source materials for both Berndt and Agarwal.

Dyson on Ramanujan's notebooks

Freeman J. Dyson, the renowned Physicist ([XVI], p.25), concludes his article entitled: *A Walk through Ramanujan's Garden*, with the following advise:

> *In conclusion, I would like to urge all of you who are working in the many fields of mathematics which have been enriched by Ramanujan's ideas to go back to the source the collected papers and the notebooks. ... The notebooks ... are now appearing in a splendidly annotated version edited by Bruce Berndt [VIII]. The "lost" notebook is now accessible to us through the devoted labors of George Andrews ([53], [56]). When I started my walk through Ramanujan's garden 47 years ago, only the collected papers were available. A year after I chose Hardy and Wright's "Theory of Numbers" (Oxford, Clarendon Press, 1938) as a school prize, I won another prize. For the second prize I chose Ramanujan's collected papers. The collected papers have traveled with me from England to America and are still as fresh to-day as they were in 1940. Whenever I am angry or depressed, I pull down the collected papers from the shelf and take a quiet stroll in Ramanujan's garden. I recommend this therapy to all of you who suffer from headaches or jangled nerves. And Ramanujan's papers are not only a good therapy for headaches. They also are full of beautiful ideas which may help you to do more interesting mathematics.*

Certainly several of us will not find this therapy useful. Since Dyson, Erdös and Selberg were among the very best in their chosen fields, they were greatly benefited by an early introduction to the Collected Papers. The rest can derive the benefit of the extensive study of Ramanujan's work by Hardy [IV] and Berndt [VIII] to begin appreciating the prolific, original work of the mathematical genius in his published work and in his notebooks.

4. Hardy on Ramanujan

In Mathematics, Hardy on Srinivasa Ramanujan is akin to Boswell on Samuel Johnson, in literature. The role of G.H. Hardy, Professor of Mathematics at the Trinity College, Cambridge, in the life and career of Ramanujan is beyond praise. Hardy considered Ramanujan was, in a way, his discovery, though he has stated that *like all other great men, he invented himself.* Ramanujan was considered a treasure and in Hardy's own words [IV],

> *I still know more about Ramanujan than any one else, am still the first authority on this particular subject. ... I owe more to him than to any one else in the world with one exception, and my own association with him is the one romantic incident in my life.*

Their association started with that historic letter of Ramanujan to Hardy, in 1913, and we have, in Chapter 1, already quoted Hardy's reactions to it. Hardy had seen Ramanujan almost everyday for nearly five years between 1914 - 1919 and was being shown *half a dozen new results every day.* Hardy was thus in a unique position to observe and understand Ramanujan at close quarters. He has also been very generous in doing his very best to propagate the mathematics of Ramanujan through lectures and articles, after Ramanujan died in 1920. This chapter draws heavily on Hardy's views of Ramanujan and quotations of Hardy are from his *NOTICE* in the *Collected Papers* [III] or from his *Ramanujan* [IV].

Hardy was not the only or even the first eminent mathematician to whom Ramanujan wrote and appealed. In Chapter 1 the response of Prof. Hill to Ramanujan's work on divergent series has been pointed out. It is hard to imagine what would have happened if Hardy, like two other countrymen of his [11], also returned the manuscript of Ramanujan without comment. In view of the pivotal role Hardy played in the career of Ramanujan, it is necessary to give a biographical account of Hardy's life and work. This is a part of the objective of this chapter.

Hardy: a brief biography

Godfrey Harold Hardy was born on 7 Feb. 1877, at Cranleigh, Surrey, in a gifted and mathematically inclined family. His father, Isaac Hardy, was an Art Master, Bursar and House Master of the preparatory branch of Cranleigh

School and his mother, Sophie Hardy, was a senior mistress at the Lincoln Training College for teachers. It is said that he was writing down numbers up to millions – his earliest sign of mathematical ability – when he was just two years old. He and his slightly younger sister were sent to a Victorian nursery and later to the Cranleigh School. At School, Hardy had private coaching in Mathematics, and not in a class, because of his precocity. He was first in his class in all subjects, but extremely shy to receive the prizes due to his self-consciousness. In Hardy's own words [XII],

> *I thought of mathematics in terms of examinations and scholarships: I wanted to beat other boys and this seemed to be the way in which I could do so most decisively.*

Though frail and good looking, he did not like to see himself in a mirror or be photographed. At the age of twelve, Hardy won a scholarship at Winchester public school, the best mathematical school in England then. He left the school with honours and was offered an open scholarship to Trinity College in Cambridge. He was not only good at studies but was also a natural Tennis player and a batsman in Cricket – and could enjoy playing or watching till almost the end of his life.

By the time he finished his schooling, he was an atheist though he came from an orthodox religious family and he stopped going into any chapel. This created problems for him at Trinity College, since attending the chapel was compulsory. He referred to God as his personal enemy all through his life. His strong feelings about the church and orthodoxy are clearly reflected in Hardy's statement:

> *It's rather unfortunate that some of the happiest hours of my life should have been spent within sound of a Roman Catholic church.*

The church referred to by Hardy is the one close to Fenner's, the cricket ground of Cambridge University.

Hardy, who graduated in 1899, managed to come as fourth Wrangler[1] in Part I of the Mathematical Tripos examination of the Cambridge University, in 1898, by any measure a superb performance, which determined the Wrangler

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[1]The top candidates were ranked as Wranglers and Wrangler meant in practical terms a First Class. An excellent account of the Tripos and the Wrangler ranking can be found in Kanigel [X].

standing. Hardy appeared for Part II of the Tripos, the more provocative and more challenging part, in 1900, and secured the first division of the first class and was elected in 1900 to a Prize Fellowship at Trinity, at the age of 22. He won the Smith's prize in 1901. It was during his coaching classes for the competitive Mathematical Tripos examination, that his coach, Professor A.E.H. Love, advised him to read Jordan's famous *Cours d' Analyse* (the first volume of which was published in 1882 and the third and last volume was published in 1887) about which Hardy said

> *I shall never forget the astonishment with which I read that remarkable work, the first inspiration for so many mathematicians of my generation, and learned for the first time as I read it what mathematics really meant. From that time onward I was in my way a real mathematician, with sound mathematical ambitions and a genuine passion for mathematics.*

In later years, Hardy traced the root of Britain's mathematical backwardness to the Tripos system which instead of remaining a means to a modest end became an end in itself and he had a major role in the reform of the Cambridge Mathematical Tripos Part I and for the abolition of the practice of the publication of the results in order of merit. In 1903, Hardy was named an M.A., which at English Universities was the highest academic degree. He became a Lecturer in Mathematics at Trinity College, in 1906. At about this time, he published his first Cambridge tract on *The Integration of Functions of a single variable.* His book *A Course of Pure Mathematics* was first published in 1908. This text book provided the first rigorous exposition of difficult mathematical concepts. It has since gone through several editions and what is more has been translated into several languages.

At the age of 33, in 1910, Hardy was elected to the Fellowship of the Royal Society (F.R.S.) of London, for his early papers since 1900, which also won for him an international reputation. The University of Cambridge recognized the growing stature and reputation of Hardy for his researches in Analysis by conferring the honorary title of Cayley Lecturer on him. He was a bachelor and was financially comfortable and research, he said, was *the one great permanent happiness of my life.* The company he had in Trinity College, Cambridge, at that time included Bertrand Russell, Whitehead, Trevelyan and Bloomsbury. Hardy considered himself a disciple of Russell not only for

his Mathematical philosophy but also for his political views and they both were pacifists who hated war. In the words of C.P. Snow :

> *In a brilliant circle, he was one of the most brilliant young men – and, in a quiet way, one of the most irrepressible.*

Ramanujan's attention was drawn to Hardy's Cambridge tract on *Orders of Infinity*, published in 1910, by Professor P.V. Seshu Aiyar, who was then at the Presidency College, Madras, when Ramanujan approached the Professor with some results in Number Theory. On page 36 of this tract, Hardy had stated that *no definite expression has as yet been found for the number of primes less than any given number*. Ramanujan told Seshu Aiyar that he had the required expression unknown to Hardy. This made Seshu Aiyar suggest to Ramanujan that this and other results may be communicated to Prof. Hardy. Thus, in January 1913, when Ramanujan wrote to Hardy for the first time, Hardy was well established as a mathematician and he had the stature of a mathematical colossus, who had a purpose in his life: to bring rigour into British Mathematical Analysis. In his biography of Ramanujan, Kanigel [X] has vividly detailed the state of English mathematics and the turn given to it by Hardy:

> *England slept in the dead calm of its Tripos system, where Newton was enshrined as God, his Principia Mathematica the Bible ... Cambridge became a school that was self-supporting, self-content, almost marooned in its limitations ... generations of English mathematicians boycotted Europe – stead-fastly clung to Newton's awkward notational system, ignored mathematical trails blazed abroad*

in Paris, Göttingen and Berlin. The first winds of change started by the end of the 19th century. Hardy chartered a course to the future and became a spokesman for the new rigour.

Hardy assisted Ramanujan (a failed F.A. of Madras University) to the Cambridge University's B.A. degree, by research, in 1916, for his long paper on Highly Composite Numbers. After the first three years of uninterrupted research activity, when Ramanujan fell ill in the summer of 1917, and never really recovered after that, and was subject to bouts of depression, Hardy strove hard to convince the concerned and succeeded in getting Ramanujan elected for the two coveted distinctions he himself had – the Fellowship of the

Royal Society and a Trinity College Fellowship – in February and October 1918, respectively.

The war years were bearable for Hardy due to the highly stimulating presence of Ramanujan. The complex circumstances in which Bertrand Russell was dismissed from his lecturership, contributed to Hardy's unhappiness at Cambridge. His Cambridge Tract on *The General Theory of Dirichlet Series*, with M. Riesz, was published in 1915. He was elected to the Savilian Chair of Geometry at Oxford University, in 1919, and so migrated to New College, where he was admired and esteemed for his mathematical talent and the conversational games which he loved to play, keeping the listener guessing as to what his real opinions were on any subject. He returned to Cambridge in 1931 to occupy a senior mathematical chair, as Sadleirian Professor, till he retired in 1942. It was at this time, his second innings at Cambridge, that C.P. Snow ran into him. In his poignant Foreword to Hardy's *A Mathematician's Apology*, published with the 1967 edition, some 27 years after it was first published, Snow starts by stating the reaction of the Cambridge mathematicians to Hardy's return to their fold:

> *They were delighted to have him back: he was a* real *mathematician, they said, not like those Diracs and Bohrs the physicists were always talking about: he was the purest of the pure ... in later days they would have said that in some indefinable way he had star quality.*

Hardy liked all ball games, played good tennis but his passion was for cricket. Till 1939, he captained the New College Senior Common Room cricket team. Hardy was a cricket afficionado, whose highest accolade (ref. Kanigel [X]), was to rate a mathematical proof as being in the *Hobbs class.* However, C.P. Snow who owed the formation of his friendship with Hardy to their common interest in cricket and who wrote on Hardy's affection for cricket till the very end of his life in an article entitled *A mathematician and cricket* (published in the Saturday Book, 8th year) states that this wasn't true. For, though Hobbs was one of Hardy's pets, Hardy wrote Snow a postcard [10] once stating:

> *Bradman is a whole class above any batsman who ever lived: if Archimedes, Newton and Gauss remain in the Hobbs class, I have to admit the possibility of a class above them, which I find difficult to imagine. They had better be moved from now on into the Bradman class.*

This is as it ought to be! For, though Bradman lost the best part of his cricketing years from international cricket due to the second World War (1940 – 45), on the question of Batting Average alone, Bradman's 99.94[2] is way above the next best batsman's, which is the Australian S.G. Barnes' 63.05, significantly above Hobbs's 56.94!

Hardy was awarded the Royal Medal of the Royal Society in 1920. He was the Secretary of the London Mathematical Society for several years and was its President for two two-year terms. The De Morgan Medal was awarded to him, in 1929 and the Sylvester Medal, in 1940. Honorary degrees were conferred on him from the Universities of Athens, Birmingham, Edinburgh, Harvard, Manchester, Marburg, Oslo and Sofia. He was an Honorary Member of many of the leading foreign Science Academies.

In 1939, Hardy had a coronary thrombosis and after that, though he recovered, his physical activities like playing tennis, cricket and squash, were over. The second world war contributed to his despair. To enliven his spirits, he was induced by friends, including Snow, to write the story of *Bertrand Russell and Trinity* – a book Hardy printed for private circulation in 1942.

Hardy wrote *A Mathematician's Apology* (1940), a brief autobiographical treatment, wherein he discusses and illustrates the attractive force of mathematics, while dismissing its utility. For, to Hardy mathematics with its depth

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[2] *Consider Sir Donald Bradman's 6996 runs for Australia at 99.94; surely he would have scored another 4 to make it 7000 at 100 if he had realized it would give him two round figures – or is it better as it is? Less well known is the fact that Albert Trott had a batting average of 102.50 for Australia, when unexplicably left out of the Australian touring team of 1896, he came here [England], joined Middlesex and retains that remarkable Test average*, states John Arlott in his Preface to "Test Cricket Records from 1877", compiled by James Gibbs (Collins, London, 1979). However, in Gibbs' compilation A.E. Trott's average does not find a mention under the 'Best batting averages', where the qualification to be included is set as a minimum aggregate of 1000 runs in Tests. Trott played only in 3 matches, 5 innings and since he was thrice not out his aggregate of 205 (including 85 not out as his highest score in an innings) gave him the 'remarkable Test average' of 102.5! On the other hand, Bradman played 52 matches (as a captain in 24 of them), 80 innings, was ten times not out and made an aggregate of 6996 runs (including 29 centuries and thirteen 50's), with a heighest score of 334 in an innings and averaged 99.94. Hence, he stands at the top of the list of best batting averages, in Gibbs's compilation!

and beauty is a creative art. In it he states:

> *I have never done anything 'useful'. No discovery of mine has made, or is likely to make, directly or indirectly, for good or ill, the least difference to the amenity of the World.*

In the context of Pure versus Applied mathematics, Hardy wrote:

> *It is true that there are branches of applied mathematics, such as ballistics and aerodynamics, which have been developed deliberately for war ... but none of them has any claim to rank as "real". They are indeed repulsively ugly and intolerably dull; even Littlewood could not make ballistics respectable, and if he could not, who can?*

E.C. Titchmarsh in his biographical article at the beginning of the *Collected Papers of G.H. Hardy* [XIII] (edited by a committee of mathematicians appointed by the London Mathematical Society and published in seven volumes (1966 – 1979)) draws attention to the one exception to this statement of Hardy, viz. *Hardy's law* based on a contribution of Hardy to genetics, which is of central importance in the study of Rh blood groups. In this context, it is also to be noted that Hardy became a Fellow of the Astronomical Society, in 1918 to participate in the expositions of Sir Arthur Eddington and in 1930, he even took part in a debate on Stellar Structure, which involved R.H. Fowler's work on Emden's and allied differential equations.

In the *Apology* Hardy spells out what ought to be the dominant motivations for pursuing research. He states

> *...intellectual curiosity, desire to know the truth. Then, professional pride, anxiety to be satisfied with one's performance, the shame that overcomes any self-respecting craftsman when his work is unworthy of his talent. Finally, ambition, desire for reputation, and the position, even the power or the money, which it brings. ...If intellectual curiosity, professional pride, and ambition are the dominant incentives to research, then assuredly no one has a fairer chance to gratifying them than a mathematician. ... as history proves abundantly, mathematical achievement, whatever its intrinsic worth, is the most enduring of all. ... the noblest ambition is that of leaving behind something of permanent value.*

Like his text book *A Course of Pure Mathematics*, his *Mathematician's Apology*, first published by the Syndics of the Cambridge University Press in 1940, and reprinted later from 1967 with a Foreword by C.P. Snow, became a classic. Generations of mathematicians have considered this short essay of Hardy as a masterpiece to be treasured.

Besides the books already referred to, Hardy published, with E.M. Wright, a book on *The Theory of Numbers* (1934) and, with W.W. Rogosinski, a book on *Fourier Series* (1944). His last book was on *Divergent Series*, completed but not published at the time of his death. He was instrumental in founding of the Quarterly Journal of Mathematics (Oxford series). In the *Collected Papers of G.H. Hardy* [XII], the list of publications given at the end lists 404 contributions including about 25 Questions covering every kind of analysis in: Diophantine Approximation, Additive Number Theory, Multiplicative Number Theory, Inequalities, Trigonometric series, mean values of power series, special functions, theory of functions, integral calculus, theory of series, integral equations and integral transforms. His work had a profound influence throughout the whole of analysis and on modern mathematics. Indeed he realized his life's *noblest ambition*, that of leaving behind something of permanent value.

After his retirement in 1942 from Cambridge University, his physical health deteriorated by the mid 40s and in the summer of 1947 he fell seriously ill and at this time in his life, only his devoted sister was close to him. Hardy even attempted suicide by consuming an overdose of barbiturates. Cricket still interested him. Snow [9] has said that there are only five snapshots of Hardy in existence. Four of these can be found in the first four parts of the *Collected Papers of G.H. Hardy*, (published in 1966-69) and one of these (in part II) depicts him watching an Oxford versus Cambridge Rugby football match at Cambridge (on 13 Dec. 1941). Some months before his death, Hardy was elected *associé étranger* of the Paris Academy of Sciences – a particular honour, since there are only 10 of these from all nations and scientific subjects, at any given time. A couple of weeks before his death, the Royal Society announced that their highest honour, the *Copley Medal* would be conferred on him. Hardy who was well known for his sarcastic sense of humor is supposed to have said on receiving this communication:

> *Now I know that I must be pretty near the end. When people hurry up to give you honorific things there is exactly one conclusion to be drawn.*

He himself had hurried the then President of the Royal Society to confer the F.R.S. on Ramanujan when the latter was repeatedly being confined to sanatoria. Death came to him suddenly and quietly, in the early hours of Dec. 1, 1947. He did not live to receive the Copley Medal, which was to have been conferred on that day, just as Ramanujan did not return to Cambridge to take up the Trinity Fellowship.

Hardy is the second hero in Kanigel's *The Man Who Knew Infinity: A Life of the Genius Ramanujan* [X]. Kanigel portrays Hardy as scrupulously honest, fastidious about giving others their due; ever so kind to the weak ones; and one in whom

> *coexisted a stern, demanding streak with an indulgent liberal-mindedness, a formidable and forbidding exterior with a soft and fragile core.*

Though Hardy was the *best and truest* friend Ramanujan ever had, Kanigel [X] expresses his view that Hardy never really knew Ramanujan. The aloof, formal nature of their friendship is best illustrated by the fact that in one note written by Ramanujan to Hardy from a sanatorium, nearly three years after working together, Ramanujan explains that Srinivasa in his name is not his first name but a part of his father's name. The richness and extent of Ramanujan's inner spiritual life was, perhaps, something he never revealed to Hardy.

In what could be called as a larger than life portrayal, in Kanigel's biography of Ramanujan [X], Hardy emerges as an aristocrat of the intellect; busy being brilliant; effectively inculcating in Ramanujan the Gospel of proof and rigour in mathematics; the embodiment of all that was highest and best in the Western mathematical tradition; the most stern taskmaster; the most generous man with not even a wisp of envy tainting his relationship with Ramanujan; who championed and hailed the gifts of Ramanujan, recognizing Ramanujan's genius and pushing it towards its limits.

Hardy's collaborations

Hardy had two fruitful collaborations in mathematics in his life. He always considered J.E. Littlewood as a better mathematician than himself[3] and Ramanujan as a natural, unorthodox, mathematical genius. The Hardy–Ramanujan collaboration overlapped with the war years but continued uninterrupted for three years between 1914 - 1917, and ended with Ramanujan's departure to India, in early 1919. During those five years they wrote five papers of the highest class which revealed that *Ramanujan Intuition incarnate, had run smack into Hardy the Apostle of Proof*, (Kanigel [X]). In the words of Littlewood, the collaboration was between two mathematicians *of quite unlike gifts, in which each contributed the best, most characteristic, and most fortunate work that was in him.*

The Hardy–Littlewood collaboration started in 1911 and barring the interference due to the war years when Littlewood was away (for four years), doing ballistics as a Second Lieutenant in the Royal Artillery, this partnership lasted 35 years. Together they published nearly a hundred papers and in the words of C.P. Snow [10], *a good many of them 'in the Bradman class'* and it is

> *the most famous collaboration in the history of mathematics... The Hardy–Littlewood researches dominated English pure mathematics, and much of world pure mathematics, for a generation... Of its enduring value there is no question... But no one knows how they did it... No one has any evidence how they set about it,*

since during their most productive period, they were not even at the same university. Hardy with Littlewood and G. Pólya published a book on *Inequalities* in 1934. Littlewood and Hardy conducted a seminar or *conversation class* at Cambridge which attracted the mathematical community due to its informal nature and openness in discussions.

Hardy was a confirmed bachelor, in whose life no woman other than his mother and sister played the slightest substantive role. He was a member of a secret intellectual Society called Apostles along with some of the

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[3]See Hardy's self-assessment and relative rating of Littlewood, Hilbert and Ramanujan, p.73 of this work.

most brilliant Cambridge men of that period – Bertrand Russell, Whitehead and James Clerk Maxwell. A homosexual current ran through the Society. Kanigel [X] asserts that the Hardy–Ramanujan relationship was marked by distance, not comradely intimacy and that Mathematics was the common ground of their relationship. A fact which indirectly corroborates this view is that after his return to India in April 1919, during the entire period of almost 13 months before he died, Ramanujan wrote only one letter to Hardy and that too in January 1920. Even in that letter he did not mention about his deteriorating health. He only wrote about the new results he had on what he called as *mock* theta functions. This led Hardy to feel, as he stated later on, that Ramanujan was recovering. So, the announcement of Ramanujan's death in a letter by his brother to Hardy, came as a shock to the latter.

We have already discussed the Hardy–Ramanujan collaboration which lead to five papers together and the 'astonishing theorem' on $p(n)$. The best in Ramanujan was brought out by Hardy [IV, p.10]:

> *The great bulk of Ramanujan's published work was done in England. His mind had hardened to some extent, and he never became at all an "orthodox" mathematician, but he could still learn to do new things, and do them extremely well. It was impossible to teach him systematically, but he gradually absorbed new points of view. In particular he learnt what was meant by proof, and his later papers, while in some ways as odd and individual as ever, read like the works of a well-informed mathematician. His methods and weapons, however, remained essentially the same.*

Hardy did perhaps everything which one human being could have ever done to another to keep Ramanujan in the best of spirits – the Bachelors Degree in Arts, the Fellowships of the Royal Society and the Trinity College, conferred on Ramanujan were mainly due to the untiring efforts of Hardy. Together Hardy and Ramanujan were a formidable mathematical pair who left indelible footprints in the sands of pure mathematics – a unique pair in the realm of Mathematics.

Pros and cons

Hardy has been eloquent to state that Ramanujan *was by far the greatest formalist of his time*; that Ramanujan's work in the fields of identities and

continued fractions is a masterpiece and he wrote that his *extremely strong language*, in the following paragraph [IV, p.14], *is not extravagant*:

> *It was his insight into algebraical formulae, transformations of infinite series, and so forth, that was most amazing. On this side most certainly I have never met his equal, and I can compare him with only Euler or Jacobi. He worked, far more than the majority of modern mathematicians, by induction from numerical examples; all his congruence properties of partitions, for example, were discovered in this way. But with his memory, his patience, and his power of calculation he combined a power of generalization, a feeling for form, and a capacity for rapid modification of his hypotheses, that were often really startling, and made him, in his own peculiar field, without a rival in his day.*

On Ramanujan's work on the Prime Number Theorem, Hardy states:

> *All that is plain is that Ramanujan found the form of the Prime Number Theorem for himself. This was a considerable achievement; for the men who had found the form of the theorem before him, like Legendre, Gauss, and Dirichlet, had all been very great mathematicians; and Ramanujan found other formulae which lie still further below the surface.* ...([IV], p.19).

On the importance of Ramanujan's work, Hardy states [IV, p.7]:

> *Opinions may differ about the importance of Ramanujan's work, the kind of standard by which it should be judged, and the influence which it is likely to have on the mathematics of the future. It has not the simplicity and the inevitableness of the very greatest work; it would be greater if it were less strange. One gift it shows which no one can deny, profound and invincible originality. He would probably have been a greater mathematician if he could have been caught and tamed a little in his youth; he would have discovered more that was new, and that, no doubt, of greater importance.*

In a report submitted to the University of Madras on Ramanujan's Mathematical work in England, he states:

> *In one respect Mr. Ramanujan has been most unfortunate. The war has naturally distracted three-quarters of the interest that*

would otherwise have been taken in his work, and has made it almost impossible to bring his results to the notice of the continental mathematicians most certain to appreciate it. It has moreover deprived him of the teaching of Mr. Littlewood, one of the great benefits which his visit to England was intended to secure. All this will pass; and, in spite of it, it is already safe to say that Mr. Ramanujan has justified abundantly all the hopes that were based upon his work in India and has shown that he possesses powers as remarkable in their way as those of any living mathematician. His work is only the more valuable because his abilities and methods are of so unusual a kind, and so unlike those of a European mathematician trained in the orthodox school.

Hardy concludes this report with the following comments:

My account of Mr Ramanujan's work has been necessarily fragmentary and incomplete. I have said enough, I hope, to give some idea of its astonishing individuality and power. India has produced many talented mathematicians in recent years, a number of whom have come to Cambridge and attained high academical distinction. They will be the first to recognize that Mr Ramanujan's work is of a different category. In him India now possesses a pure mathematician of the first order, whose achievements suggest the brightest hopes for its scientific future.

The above statements of Hardy are all positive and strong and bordering on adulation. To show that Hardy was objective in his assessment of Ramanujan, a few of the drawbacks of Ramanujan pointed out by Hardy are given below:

One would have thought that such a formalist as Ramanujan would have revelled in Cauchy's Theorem, but he practically never used it, and the most astonishing testimony to his formal genius is that he never seemed to feel the want of it in the least. (Hardy [IV, p.10])

... in the analytic theory of numbers he had, in a sense, discovered a great deal, but he was a very long way from understanding the real difficulties of the subject. ... in analytic theory of numbers,

even Ramanujan's imagination led him seriously astray. ... he showed, as always, astonishing imaginative power, but he proved next to nothing[4]*, and a great deal even of what he imagined was false.* (Hardy [IV, p.10,16])

He had no knowledge at all, at any time, of the general theory of arithmetical forms. (Hardy ([IV, p.11])

In analysis proper Ramanujan's work is inevitably less impressive, since he knew no theory of functions, and you cannot do real analysis without it... Still, Ramanujan rediscovered an astonishing number of the most beautiful analytic identities. Thus the functional equation for the Riemann Zeta-function ... stands (in an almost unrecognizable notation) in the notebooks. So does Poisson's summation formula ... and so also does Abel's functional equation. (Hardy [IV, p.14])

Ramanujan knew nothing at all about the theory of analytic functions. ... Ramanujan's theory of primes was vitiated by his ignorance of functions of a complex variable. It was (so to say) what the theory might be if the Zeta-function had no complex zeros. His method depends upon a wholesale use of divergent series ... That his proofs should have been invalid was only to be expected. But the mistakes went deeper than that, and many of the actual results were false. He had obtained the dominant terms of the classical formulae, although by invalid methods; but none of them are such close approximations as he supposed. ... This may be said to have been Ramanujan's one great failure. (Hardy [IV, p. 19-20])

Hardy thus was not only lavish in his appreciation of the originality and the mathematical genius of Ramanujan, but also harsh and severe in his criticism of the shortcomings of Ramanujan. While he himself edited only one chapter on the hypergeometric series from the second notebook of Ramanujan, he estimated that

about two-thirds of Ramanujan's best Indian work was rediscovery,

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[4]Hardy's reference is to the work of Ramanujan before he went to Cambridge.

> *and comparatively little of it was published in his lifetime, though Watson, who has worked systematically through his notebooks, has since disinterred a good deal more.*

This estimate has been refuted to be too high by Bruce C. Berndt, who studied each and every one of the 3254 formulae of Ramanujan in his notebooks, over a period of two decades and to understand this one has to study the five Part magnum opus of Berndt [VIII].

Hardy in his *Apology* has stated that all his best work, after he began his life long collaboration with Littlewood in 1911, and after he 'discovered Ramanujan' in 1913,

> *has been bound up with theirs, and it is obvious that my association with them was the decisive event in my life. I still say to myself when forced to listen to pompous and tiresome people, 'Well, I have done one thing* you *could never have done, and that is to have collaborated with both Littlewood and Ramanujan on something like equal terms'.*

Lectures on Ramanujan's work

Hardy who devoted several years of his life-time to the study of the work of Ramanujan, gave many lectures on Ramanujan's work at a number of universities and societies in America and England, as well as courses of lectures at Princeton[5] and Cambridge. In the *Apology* he categorically states:

> *I hate 'teaching', and have had to do very little, such teaching as I have done having been almost entirely supervision of research; I love lecturing, and have lectured a great deal to extremely able classes; and I have always had plenty of leisure for the researches which have been the one great permanent happiness of my life.* The *matter, delivery and hand-writing (a specimen of which appears on the dust cover of A Mathematician's Apology) were alike fascinating*[6].

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[5]Notes for the Princeton series were written by Marshall Hall and these were printed as lectures notes. A copy of the same is in the Library of the University of Wisconsin, Madison, Wisconsin. (Private communication from Prof. R.A. Askey).

[6]Also quoted by Titchmarsh in the Collected papers of G.H. Hardy [XIII, Vol. I].

Hardy has done yeoman service to Mathematics by publishing these lecture courses of his as a book [IV] entitled: *Ramanujan: Twelve lectures on subjects suggested by his life and work*, in 1940. Hardy was a Visiting Professor at Princeton and at the California Institute of Technology during 1928 – 29. He delivered two lectures at the Harvard Tercentenary Conference of Arts and Sciences in the fall of 1936 – an occasion which *brought to Harvard more than twenty-five hundred scholars for lectures under broad rubrics of knowledge* ([X], p.373). On September 16, before an august audience consisting of *no fewer than eleven Nobel Prize winners* Hardy gave his lecture on Ramanujan

> *... the most romantic figure in the recent history of mathematics; a man whose career seems full of paradoxes and contradictions, who defies almost all the canons by which we are accustomed to judge one another, and about whom all of us will probably agree in one judgement only, that he was in some sense a very great mathematician.*

This Harvard lecture is reprinted from the American Mathematical Monthly, where it was first published, in the above said book on *Ramanujan.* As pointed out earlier, Hardy had lectured at *a number of universities and societies in America and England and connected courses in Princeton and Cambridge* on Ramanujan's work and the remaining 11 chapters in *'Ramanujan'* are based on these lectures of Hardy and *Ramanujan is the thread which holds the whole together.* Hardy's aim in these lectures is stated clearly in his own words, in the Harvard lecture:

> *... the reputation of a mathematician cannot be made by failures or by rediscoveries; it must rest primarily, and rightly, on actual and original achievement. I have to justify Ramanujan on this ground, and that I hope to do in my later lectures.*

The chapter titles given below indicates the range of problems on which Ramanujan had worked during his short productive period, as a professional researcher, after he came into contact with Hardy:

I. The Indian mathematician Ramanujan
II. Ramanujan and the theory of numbers
III. Round numbers

IV. Some more problems of the analytic theory of numbers
V. A lattice-point problem
VI. Ramanujan's work on partitions
VII. Hypergeometric series
VIII. Asymptotic theory of partitions
IX. The representation of numbers as sums of squares
X. Ramanujan's function $\tau(n)$
XI. Definite integrals
XII. Elliptic and modular functions

Apart from these lectures, Hardy has submitted a report on Ramanujan's work in England, for publication, at the request of the University of Madras. This report is with the Wren Library of Trinity College and has the Card Catalogue number Add.Ms.a.94[12(79)] (see Appendix 2 for details about the catalogued articles). The report gives summaries of twelve (out of the 21) papers published by Ramanujan from England, including one (of the seven) he wrote with Hardy. Ten of these are in the *Collected papers*, enumerated as 11, 12, 6, 14, 16, 12, 15, 17, 18 and 20 (see Appendix 1 for the titles of these papers). One of the 12 corresponds to 19 (of Appendix 1) though with a slightly different title: *Some series for Euler's constant* and another (with Hardy) has the title *A problem in the analytic theory of numbers*, which was unpublished at that time but communicated to the London Mathematical Society. Obviously, since this title is not one of the seven joint papers listed as 31 to 37 in the *Collected papers*, it was rewritten subsequently under a different title.

Hypergeometric series

Hardy remarks: *any student of the notebooks can see the Ramanujan's ideal of presentation had been copied from Carr's* Synopsis. After Ramanujan's death, in 1923, Hardy spent three to four months on Chapter XII[7] of the second notebook, on Hypergeometric series, and in the introduction to the paper he published on this topic he wrote: *a systematic verification of the results would be a very heavy undertaking.*

[7]Hardy states in a Note at the end of lecture VII in his *Ramanujan* that this is chapter X in Watson's copy of the 'second edition'.

This section is intended for the mathematically inclined readers. A few essential definitions are given to enable the reader to appreciate some of the results of Ramanujan in this area.

The British mathematician John Wallis (in his book, *Arithmetica Infinitorum*, London (1655)), studied the series

$$1 + a + a(a+1) + a(a+1)(a+2) + \cdots \tag{1}$$

which is a generalization of the ordinary geometric series

$$1 + a + a^2 + a^3 + \cdots \tag{2}$$

and called (1) as the *hypergeometric*[8] series. The infinite series

$$1 + \frac{ab}{c}\frac{z}{1!} + \frac{a(a+1)b(b+1)}{c(c+1)}\frac{z^2}{2!} + \cdots, \tag{3}$$

usually represented by the symbol ${}_2F_1(a, b; c; z)$ was defined by Carl Friedrich Gauss, in 1812, in his famous thesis [85] *Disquisitiones generales circa seriem infinitam*, before the Royal Society in Göttingen, Germany. He proved the famous summation theorem, since named after him as the Gauss summation theorem:

$${}_2F_1(a, b; c; 1) \;=\; \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} \tag{4}$$

where $\Gamma(x)$ is the ordinary gamma function

$$\Gamma(x) \;=\; (x-1)\Gamma(x-1) \;=\; (x-1)(x-2)\cdots 2.1 \;=\; (x-1)!$$

with $\Gamma(0) \;=\; 1$. Gauss regarded the ${}_2F_1$ as a function, in general, of four real or complex variables, rather than as a series in z, and gave a remarkably full discussion of the convergence of the series – the series is convergent for $|\, z \,| < 1$, divergent for $|\, z \,| > 1$ and for $z = 1$, it is convergent if $Rl(c-a-b) > 0$ and divergent if $Rl(c-a-b) < 0$. In ${}_2F_1(a, b; c; z)$, the parameters a and b are called as the numerator parameters, while c is called the denominator parameter. If either of the numerator parameters is a zero or a negative integer, say, $-n$, then the series (3) has only $n+1$ terms and

[8]The Greek word *hyper* means above or beyond.

it becomes a polynomial of degree n in z. If the denominator parameter c is zero or a negative integer ; or, if the zero due to the denominator parameter occurs before the zero due to the numerator parameter; then the hypergeometric series is not defined (in one of the conventions).

Conventionally, in the modern notation (due to Barnes [86] – who, in 1907, developed the integral representations for the hypergeometric functions, first introduced by Euler [84], in 1748):

$$ {}_2F_1(a,b;c;z) \;=\; \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{z^n}{n!}, \tag{5} $$

where the symbol

$$ (a)_n \;=\; \frac{\Gamma(a+n)}{\Gamma(a)} \;=\; a(a+1)(a+2)\cdots(a+n-1), \quad (a)_0 = 1, \tag{6} $$

is called the Pochammer [87] symbol, or as the shifted/raising factorial, or as a generalized power. Kummer [88], in 1836, showed that the hypergeometric function, ${}_2F_1(z)$, satisfies a second order ordinary differential equation:

$$ \frac{d^2y}{dz^2} + \left[\frac{c}{z(1-z)} - \frac{1+a+b}{1-z} \right] \frac{dy}{dz} - \frac{ab}{z(1-z)} y = 0, \tag{7} $$

from which 0 and 1 are seen to be regular singular points of the equation. If $1/z$ replaces z, then ∞ is also a regular singular point of the Gauss equation (cf. [89], §10.3). It was Kummer [88] who showed that there exist in all 24 solutions for the Gauss equation, which cover the entire complex z-plane, provided that a, b and c are not integers nor zero ([90], [91], for all the 24 solutions). The hypergeometric function has retained its significance in modern mathematics because of its powerful unifying influence, since an intimate relationship exists between the ${}_2F_1$ and the special functions of mathematical physics ([91], Theorem 9.1).

Clemsen (1828) introduced a series which is a generalization of (3), by increasing the number of numerator and denominator parameters to 3 and 2, respectively. This lead to further generalizations in this direction and summation theorems, like the Gauss summation theorem for the ${}_2F_1(1)$, have

been obtained by Saalschütz (1980), Dixon (1903) and Dougall (1907) for generalized hypergeometric series[9]. Ramanujan was not aware of these developments in this field when he was noting down his results in his notebooks.

In Chapter 1, the first four formulae (1) – (4) are specialized examples of Ramanujan's work on hypergeometric series and these were communicated in his first letter to Hardy in 1913. Ramanujan considered the hypergeometric series (in chapters X and XI of his second notebook):

$$ {}_3F_2\left[\begin{matrix}\alpha_1, \alpha_2, \alpha_3; 1\\ \beta_1, \beta_2\end{matrix}\right] = \sum_{n=0}^{\infty} \frac{(\alpha_1)_n\,(\alpha_2)_n\,(\alpha_3)_n}{(1)_n\,(\beta_1)_n\,(\beta_2)_n} $$

where $(\alpha)_n = \alpha(\alpha+1)(\alpha+2)\ldots(\alpha+n-1)$ with $(\alpha)_0 = 1$. This chapter

> *contains in a condensed form practically everything that is known about the summation and transformation of this series... It contains 47 main theorems, many of them followed by a separate statement of a number of corollaries and particular cases. Thus formula 40 is followed by 'examples' 1–18.*

Prof. S.R. Ranganathan recalls [38] a conversation with Hardy (in March 1925) wherein Hardy said:

> *This work has taken me several weeks. If I am to edit the entire notebook, it will take the whole of my life-time. I cannot do my own work. This would not be proper.*

To illustrate the work of Ramanujan in this area, we look at a few of the *curious formulae* of his, in the standardized notation of Barnes for generalized hypergeometric series:

$$ 1 + 3\frac{x-1}{x+1} + 5\frac{x-1}{x+1}\frac{x-2}{x+2} + \cdots = x \tag{8} $$

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[9]The monograph of Bailey [72] on *Generalized Hypergeometric Series* provides an excellent summary of the whole theory, from its origins, as it existed then.

Using the Horner scheme for polynomial evaluation[10] for the Generalized hypergeometric series (Srinivasa Rao and Venkatesh [65]):

$$ {}_pF_q\left(\begin{array}{l}\alpha_1,\alpha_2,\cdots,\alpha_p;\\ \beta_1,\beta_2,\cdots,\beta_q\end{array}\quad z\right)=\left[1+\frac{x_0}{y_0}\left(z+\frac{x_1}{y_1}\left(z+\frac{x_2}{y_2}\left(z+\cdots\right)\right)\right)\right] $$

where

$$ x_i=\prod_{j=1}^{\infty}(\alpha_j+i)\quad\text{and}\quad y_i=(i+1)\prod_{k=1}^{\infty}(\beta_k+i) $$

it can be easily seen that (8) corresponds to:

$$ {}_3F_2\left(\begin{array}{l}1,\frac{3}{2},-x+1;\\ \frac{1}{2},x+1\end{array}\quad -1\right)=x, $$

which is a special case of

$$ {}_3F_2\left(\begin{array}{l}a,\frac{a}{2}+1,b;\\ \frac{a}{2},1+a-b\end{array}\quad -1\right)=\frac{\Gamma(\frac{a}{2}+\frac{1}{2},1+a-b)}{\Gamma(a+1,\frac{a}{2}+\frac{1}{2}-b)} $$

for $a=1,b=-x+1$. Another series formula of Ramanujan is the alternating sum:

$$ 1-3\frac{x-1}{x+1}+5\frac{x-1}{x+1}\frac{x-2}{x+2}+\cdots=0, \tag{9} $$

a special case of Dixon's theorem specialized to terminating series:

$$ {}_3F_2\left(\begin{array}{l}a,b,-n;\\ 1+a-b,1+a+n\end{array}\quad 1\right)=\frac{(1+a)_n(1+\frac{a}{2}-b)_n}{(1+\frac{a}{2})_n(1+a-b)_n}, $$

for $a=1,b=\frac{3}{2},n=x-1$. A third formula of Ramanujan reads:

$$ 1+3(\frac{x-1}{x+1})^2+5(\frac{x-1}{x+1}\frac{x-2}{x+2})^2+\cdots=\frac{x^2}{2x-1} \tag{10} $$

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[10]The standard procedure for polynomial evaluation developed in 1819 by Horner (cf. in [92]) is to write the polynomial in a folded form:

$$ f(x)=a_nx^n+a_{n-1}x^{n-1}+\cdots+a_1x+a_0 $$
$$ =((\cdots(((a_nx+a_{n-1})x+a_{n-2})x+a_{n-3})x+\quad\cdots+a_1)x+a_0), $$

so that a minimum number of multiplications and temporary storage locations (in a computer) are used.

which corresponds to:

$$ {}_4F_3\left(\begin{array}{l} 1, \frac{3}{2}, -x+1, -x+1; \quad 1 \\ \frac{1}{2}, x+1, x+1 \end{array}\right) = \frac{x^2}{2x-1}, $$

which is a special case of the summation theorem:

$$ {}_4F_3\left(\begin{array}{l} a, \frac{a}{2}+1, b, c; \qquad\qquad 1 \\ \frac{a}{2}, 1+a-b, 1+a-c \end{array}\right) = \frac{\Gamma(\frac{a}{2}+\frac{1}{2}, 1+a-b, 1+a-c, \frac{1}{2}+\frac{a}{2}-b-c)}{\Gamma(a+1, \frac{1}{2}+\frac{a}{2}-b, \frac{1}{2}\frac{a}{2}-c, 1+a-b-c)}, $$

for $a = 1, b = c = -x+1$. Another formula reads:

$$ 1 - 3(\frac{x-1}{x+1})^2 + 5(\frac{x-1}{x+1}\frac{x-2}{x+2})^2 + \cdots = \frac{\Gamma(x+1)^2}{\Gamma(2x)} \qquad (11) $$

which corresponds to:

$$ {}_4F_3\left(\begin{array}{l} 1, 3/2, -x+1, -x+1; \quad -1 \\ 1/2, x+1, x+1 \end{array}\right) = \frac{\Gamma(x+1)^2}{\Gamma(2x)}, $$

a special case of

$$ {}_4F_3\left(\begin{array}{l} a, \frac{a}{2}+1, b, c; \qquad\qquad -1 \\ \frac{a}{2}, 1+a-b, 1+a-c \end{array}\right) = \frac{\Gamma(1+a-b, 1+a-c)}{\Gamma(1+a, 1+a-b-c)}, $$

for $a = 1, b = c = -x+1$. A fifth formula reads:

$$ 1 + \frac{x-1}{x+1} + \frac{x-1}{x+1}\frac{x-2}{x+2} + \cdots = 2^{2x-1}\frac{\Gamma(x+1)^2}{\Gamma(2x+1)}, \qquad (12) $$

which corresponds to:

$$ {}_2F_1\left(\begin{array}{l} 1, -x+1; \quad -1 \\ x+1 \end{array}\right) = 2^{2x-1}\frac{\Gamma(x+1)^2}{\Gamma(2x+1)}, $$

a special case of Kummer's theorem:

$$ {}_2F_1\left(\begin{array}{l} a, \ b; \qquad -1 \\ 1+a-b \end{array}\right) = \frac{\Gamma(1+\frac{a}{2}, 1+a-b)}{\Gamma(1+a, 1+\frac{a}{2}-b)}, $$

for $a = 1, b = 1 - x$, obtained after using the duplication formula:

$$2 + 2x\Gamma(x + \frac{1}{2})\Gamma(x + 1) = \sqrt{\pi}\Gamma(2x + 1). \tag{13}$$

If (12) is an alternating sum:

$$1 - \frac{x-1}{x+1} + \frac{x-1}{x+1}\frac{x-2}{x+2} - \cdots = \frac{2}{2x-1}, \tag{14}$$

it corresponds to a special case of Gauss's theorem:

$${}_2F_1\left(\begin{array}{ll} a, b; & 1 \\ c & \end{array}\right) = \frac{\Gamma(c, c-a-b)}{\Gamma(c-a, c-b)},$$

for $a = 1, b = 1 - x, c = 1 + x$. A variant of the above alternating sum:

$$1 - \frac{1}{3}\frac{x-1}{x+1} + \frac{1}{5}\frac{x-1}{x+1}\frac{x-2}{x+2} - \cdots = \frac{2^{4x}(\Gamma(x+1))^4}{4x(\Gamma(2x+1))^2}, \tag{15}$$

corresponds to:

$${}_3F_2\left(\begin{array}{ll} 1, \frac{1}{2}, -x+1; & 1 \\ \frac{3}{2}, x+1 & \end{array}\right) = \frac{2^{4x}(\Gamma(x+1))^4}{4x(\Gamma(2x+1))^2},$$

a special case of Dixon's theorem:

$${}_4F_3\left(\begin{array}{ll} a, b, -n; & 1 \\ 1+a-b, 1+a+n & \end{array}\right) = \frac{(1+a)_n, (1+\frac{a}{2}-b)_n}{(1+\frac{a}{2})_n, 1+a-b)_n},$$

for $a = 1, b = \frac{1}{2}$ and $n = x - 1$, obtained after using the duplication formula (13). Yet another example of this type is the result:

$$1 + (\frac{x-1}{x+1})^2 + (\frac{x-1}{x+1}\frac{x-2}{x+2})^2 + \cdots = \frac{2x}{4x-1}(\frac{\Gamma(x+1)}{\Gamma(2x+1)})^4\Gamma(4x+1), \tag{16}$$

which corresponds to:

$${}_3F_2\left(\begin{array}{ll} 1, 1-x, 1+x; & 1 \\ 1-x, 1+x & \end{array}\right) = \frac{2x}{4x-1}(\frac{\Gamma(x+1)}{\Gamma(2x+1)})^4\Gamma(4x+1),$$

and it is a special case of Dixon's theorem:

$$ {}_3F_2\left(\begin{matrix} a, b, c; \\ 1+a-b, 1+a-c \end{matrix} \; 1 \right) = \frac{\Gamma(\frac{a}{2}+1, 1+a-b, 1+a-c, 1+\frac{a}{2}-b-c)}{\Gamma(a+1, 1+\frac{a}{2}-b, 1+\frac{a}{2}-c, 1+a-b-c)}, $$

for $a = 1, b = c = 1 - x$, obtained after using the duplication formula (13). The formulae (8), (9), (15) and (16) are given in the Examples to be proved in W.N. Bailey's: *Generalized Hypergeometric Series* [72].

However, the key formula in Ramanujan's chapter XII, edited by Hardy, is:

$$ {}_7F_6\left(\begin{matrix} s, 1+\frac{1}{2}s, -x, -y, -z, -u, x+y+z+u+2s+1 \\ \frac{1}{2}s, x+s+1, y+s+1, z+s+1, u+s+1, v+s+1 \end{matrix} ; 1 \right) $$

$$ = \frac{1}{\Gamma(s+1)\Gamma(x+y+z+u+s+1)} \prod_{x,y,z,u} \frac{\Gamma(x+s+1)\Gamma(y+z+u+s+1)}{\Gamma(z+u+s+1)} $$

> *In algebra, Ramanujan's main work was concerned with hypergeometric series and continued fractions (I use the word algebra, of course, in its old-fashioned sense). These subjects suited him exactly, and here he was unquestionably one of the greatest masters. There are now three famous identities, the "Dougall–Ramanujan identity" ... and the "Rogers–Ramanujan identities", in which he had been anticipated by British mathematicians ... As regards hypergeometric series one may say, roughly, that he discovered the formal theory, set out in Bailey's tract [72], as it was known up to 1920. There is something about it in Carr, and more in Chrystal's 'Algebra', and no doubt he got his start from that. ... Ramanujan seems to have found the formula about 1910 or 1911, but he had been anticipated by Dougall. The formula looks formidable, but Dougall's proof is very simple. I imagine that Ramanujan argued similarly, but there is nothing in the notebooks to show.* (Hardy [XII], p.12 and 102).

Saalschütz's theorem (1890) for a terminating ${}_3F_2(1)$ and Dixon's theorem (1903) for any convergent well-poised series[11] of the type ${}_3F_2(a, b, c; 1+a-b, 1+a-c; 1)$, are two of the important results of Ramanujan obtained as

[11] In ${}_AF_B(a_1, \cdots, a_A; b_1, \cdots, b_B; z)$, if the sum of the denominator parameters exceeds

limiting cases of this key formula, called by Hardy ([IV], Ch. VII) as the Dougall–Ramanujan summation theorem for a well-poised ${}_7F_6(1)$.

Ramanujan seems to have never published this result and it is found only in his notebook. Most of the results in that chapter of Ramanujan can be derived from this key result.

Basic hypergeometric series were first introduced and studied by Heine who replaced the parameters a, b, c in the Gauss hypergeometric series ${}_2F_1(a, b; c; z)$ by basic numbers $\frac{1-q^a}{1-q}$ (similarly for b and c), where $0 < q < 1$ and the resultant series is called the basic hypergeometric series denoted by ${}_2\phi_1(q^a, q^b; q^c; z)$ which in the limit $q \to 1$ becomes the Gauss series. q-extensions of almost all the known ordinary and special functions of mathematical physics were obtained since Heine's basic hypergeometirc series was introduced. Bailey's [72] and Slater's [90] books are the early books which contain these extensions. The famous Rogers - Ramanujan identities can be shown ([IV], ch.VII) to follow from a series transformation formula which contains the q-extension of the very-well poised 2-balanced ${}_7F_6$ sum.

In chapters XII, XIII and XV of the first notebook and chapters X and XI of the second notebook Ramanujan studies hypergeometric series. His chapters X and XI of the second notebook have, respectively, 35 and 36 numbered entries on hypergeometric series, each one of which has been studied by Berndt ([VIII], Part II, chs. 10 and 11). Ramanujan wrote down the first few terms in each series and did not use any notation, like the later day notation of Barnes [86] for the hypergeometric series. So, it was easy for him to discern the elegance in formulas involving series. In this context, Askey's article [96] is a good starting point for the interested student who wants to learn about Ramanujan's contributions to ordinary and basic hypergeometric series.

Ramanujan obtained results on continued fractions from the three-term recurrence relations he derived for himself for the hypergeometric series. He rediscovered not only many of the classical results of Euler, Gauss and Heine

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the sum of the numerator parameters by 1, i.e. if $b_1+b_2+\cdots b_B = a_1+a_2+\cdots a_A+1$, then the series is said to be a Saalschützian. If $A = B+1$ and $1+a_1 = b_1+a_2 = b_2+a_3 = \cdots = b_B+a_{B+1}$, then the series is said to be well-poised. This nomenclature was introduced by Whipple in 1925.

and others on hypergeometric series and continued fractions but also obtained many new results. His 'Lost' notebook contains many results related to Heine's q-series and and some of the q-continued fractions are related to modular functions. In the words of Prof. K.G. Ramanathan ([55], 1987, p.278):

> *The exploitation of this relationship between modular functions and continued fractions is one of the most beautiful aspects of Ramanujan's work on continued fractions.*

Hardy tried to *annotate* one chapter from the notebooks and after that experience he decided to go back to his own reserach work and left it to G.N. Watson to take up the job of fully editing the notebooks with B.M. Wilson. However, as pointed out in chapter 3 of this book, this work had to wait for more than five decades, after the death of Ramanujan, before Bruce C. Berndt started working on them in February 1974 and devoted his full time to the notebooks from May 1977. The tireless splendid effort of Berndt resulted in the five volume publication of his on *Ramanujan Notebooks* [VIII].

5. Chandra and Ramanujan

Dr. S. Chandrasekhar (1910 - 1994), the legendary Astrophysicist, who won the 1983 Nobel Prize in Physics, with Dr. William A. Fowler, had a great admiration for Ramanujan. Ramanujan's fame had its impact and influence on him. Chandrasekhar was responsible for:

- the authentic passport photograph of Ramanujan reaching Hardy
- a role in the founding and continuance of the Ramanujan Institute
- a role in the making of the bust of Ramanujan by Paul Granlund

Dr. S. Chandrasekhar (Chandra)

Chandrasekhar, born in Lahore on 19.10.1910, was the third child of Mrs. Seethalakshmi and Mr. C. Subrahmanya Iyer, brother of Sir C.V. Raman. He was educated at home through his primary and lower secondary school years and later joined the Hindu High School, Madras, in Form IV. He came to know of the death of Ramanujan from his mother when he was just nine years old. He was a distinguished student of the Physics Honours class of Presidency College, Madras, when he came into contact with Professors Arnold Sommerfeld (1928) and Werner Heisenberg (1929) when they visited India. The books of Sommerfeld on *Atomic Structure and Spectral lines* and Compton on *X-rays and Electrons* inspired him to publish his first paper on *The thermodynamics of Compton effect with reference to the interior of the Stars*, in the Indian Journal of Physics (15 Oct. 1928). Even before he completed his studies for the B.A. (Honours) degree of the Madras University, he published several papers including one in the Proceedings of the Royal Society (London) on *Compton scattering and the new Statistics.* With that start, he continued publishing an incessant stream of research papers, some running to more than hundred pages in journals like the Proceedings of the Royal Society, Monthly Notices of the Royal Astronomical Society, the Astrophysical Journal and the Physical Review. He won Sir Arthur Eddington's book entitled: *The Internal Constitution of Stars* (Cambridge University Press, 1926), as a Prize in a Physics essay contest and this was the beginning of his life long association with Eddington. In 1930, he was offered a special scholarship to continue his studies and research at Trinity College, Cambridge,

where he worked with Professors Ralph H. Fowler, his Ph.D. Supervisor and Paul Dirac and Milne. The award of the 'Sheep-Shanks' Prize was an early recognition of his abilities as an Astrophysicist. His was a meteoric rise to the pinnacle of glory marred by just one unfortunate incident which we now detail.

Chandrasekhar studied the problem of what happens to a star in its final stages of evolution in great detail. Stars vary greatly in size and luminosity. They range from those about the size of our planet to those hundreds of times larger than the Sun. The star nearest to the Earth is the Sun and compared to others it is a medium-sized star. Its mass and radius are the yardsticks for measuring the sizes of other stars, which are classified according to their surface temperature or colour. The smallest star known, a white dwarf star, is only about 1200 miles in diameter and has a density of nearly 1000 tons per cubic inch. This enormous density was explained, in 1925, by Prof. Ralph H. Fowler to be due to the tight packing of free electrons and ionized nuclei under extreme gravitational pressure referred to as a state of degenerate matter. Eddington's perfect gas model of the stars led him to believe that the white dwarf stage is the terminal stage in the evolution of *all* stars. It was at this juncture that Chandrasekhar took into account the special relativistic variation of mass with velocity and Fermi-Dirac statistics to arrive at the *exact* equation of state for a completely degenerate gas. The predictions of this theory led him to the *Chandrasekhar limit – 1.44 times the solar mass – for stars in their last stage of evolution, a detail worked out by him on his long voyage to Trinity College, Cambridge, in 1930.* He derived the dramatic result that stars with a mass greater than 1.44 times the solar mass, would eventually collapse past the white dwarf stage into an object of enormous density and speculated the existence of (what are called today as) extremely dense neutron stars and black holes, discovered many years later. This is an over-simplified account of an extremely careful and beautiful state-of-the-art analysis of stellar evolution, by Chandrasekhar.

Chandrasekhar's confidence (based on the irrefutable mathematical theory) however had to confront the established authority of Sir Arthur Eddington who not only refused to accept the result but felt intuitively : "*Your* [*Chandrasekhar's*] *mathematics may be all right but I don't think your Physics is correct*". The human side of this episode is recorded in the biography

of "Chandra" by Kameshwar Wali [67] and in the extremely well-written vignette for the Physics student : "Chandrasekhar and his limit" by G. Venkataraman [68].

When Chandrasekhar wanted to present his results, before the Royal Astronomical Society in London, Eddington arranged for a 30 min. presentation, instead of the customary 15 min., on Jan. 11, 1935 but scheduled for himself a talk too. After Chandrasekhar's presentation, Eddington started with :

> *I do not know whether I shall escape from this meeting alive, but the point of my paper is that there is no such thing as relativistic degeneracy (!) – Dr. Chandrasekhar had got this result before, but he has erased it in his last paper; and, when discussing it with him, I felt driven to the conclusion that this was almost a* reductio ad absurdum *of the relativistic degeneracy formula. Various accidents may intervene to save the star, but I want more protection than that. I think there should be a law of Nature to prevent a star from behaving in this absurd way.*

Eddington literally tore Chandrasekhar's work to shreds with jokes and rhetoric and ridiculed the result and Chandrasekhar himself later said about this incident :

> *I had gone to the meeting thinking that I would be proclaimed as having found something very important. Instead, Eddington made a Fool of me. I was distraught. I did not know whether to continue my career.*
>
> *I returned to Cambridge late that night, probably around one o'clock. I remember going into the common room. There was still a fire burning, and I remember standing in front of it and repeating to myself, 'This is how the world ends, not with a bang, but with a whimper.'*

Chandrasekhar's work was appreciated soon by several peers – Bohr, Pauli, Fowler, Wilson – and Rosenfeld, who read the comments of Eddington, said:

> *After having courageously read Eddington's paper twice, I have nothing to change in my previous statements, it is the wildest nonsense.*

As the controversy was raging, Chandrasekhar took a decision to ignore it after asking for himself :

> *Am I going to continue the rest of my life fighting . . . or change to other areas of interest. I said, well, I will write a book and then change my interest. So, I did.*

In his book *An Introduction to the study of Stellar Structure*, published in 1939, he asserts :

> *Since the theory is a straight forward consequence of the quantum mechanics and, further, uses Dirac's theory of the electron only in that phase of its application which has been confirmed by laboratory experiments (Klein-Nishina formula, production of Cosmic ray showers, etc.), there can be little doubt that it is essentially correct.*

The confrontation between Eddington and Chandrasekhar on this work was hotly debated for several years and at several international conferences. In 1939, at a Paris Conference on White Dwarfs and Super Novae, Chandrasekhar got an opportunity to state openly that Eddington was wrong. This historical controversy hindered the progress of research on Neutron Stars and the collapse of stars for 3 to 4 decades. It is also a rare instance where Eddington went astray in his judgement based on intuition. Chandrasekhar's theory was completely vindicated, by the 1950's, and hundreds of white dwarf stars are known today whose measured masses fall neatly on the now famous Mass-Radius relationship curve predicted by him. No white dwarf star has been found with a mass greater than 1.44 times the solar mass.

In October 1982, on the occasion of Eddington's birth centenary, Chandrasekhar was invited by the Cambridge University to deliver a series of lectures. His lectures were entitled *Eddington: The Most Distinguished Astrophysicist of His Time.* In these two lectures [69] which are great classic masterpieces, Chandrasekhar is totally objective in his assessment of the scientific works of Eddington – the astrophysicist who could be considered Chandrasekhar's greatest scientific "adversary". There was never any rancour between Eddington and Chandrasekhar despite the controversy!

The announcement of the award of the Nobel Prize in Physics, to Dr. Subrahmanyan Chandrasekhar, along with another American Physicist William A. Fowler of the California Institute of Technology, was made on the occasion of his 73rd birthday – Oct. 19, 1983 – for the discovery of the Chandrasekhar limit.

Chandrasekhar wrote several books and gave several lectures in his inimitable, impeccable style[1]. He won innumerable awards. The citation for the Padmabhushan award, in 1968, read :

> *Dr. Chandrasekhar not only played a leading role in modern India's contribution to scientific research, but is also a link between India and the West.*

Many wonder as to what has contributed to Chandrasekhar's astounding productivity over 50 years, in the light of Thomas Huxley's remark, that a man of sixty in science does more harm than good! We can only quote Chandrasekhar's own observation on Lord Rayleigh, the 19th century physicist, who remained steadily productive in a variety of fields for five decades and turned out some of his best known work (such as the discovery of the Argon gas) in the later part of his career :

> *his influence was enormous because he added to the great body of knowledge, constantly inventing many things that were not spectacular but were always important.*

This is equally true of Chandrasekhar himself, who effortlessly switched fields and worked as hard as he only could until he had the *personal satisfaction* of having gained a clear *perspective* of the field. He continued to enrich the field of Astrophysics and had a quest for *Truth and Beauty* and elegance and mathematical order, throughout his life.

On Ramanujan

On every one of his visits to India, ever since July 1936, when he first met Mrs. Ramanujan and acquired the passport photograph of Ramanujan for Hardy, he made it a point to call on the lady. However, Chandrasekhar was seldom willing to talk about Ramanujan or his work on any occasion. In 1987, on the occasion of the birth centenary celebrations of Ramanujan, at the International Conference organized at Madras, he was requested to give the first lecture. Surprisingly, he chose to speak on Newton's *Principia*, which was the topic of study for him during the last decade of his life. This was probably because he had already spoken *On Ramanujan* at the Banquet

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[1]It is said that he spoke in paragraphs and that he read his special named lectures such as the Eddington Centenary Lectures.

on June 3, 1987, at an International Conference to mark Ramanujan's birth centenary, held at the University of Illinois, Urbana–Champagne. So, it is relevant for us to reproduce here, the following remarks made by Dr. S. Chandrasekhar then[2]:

On Ramanujan[3]

S. Chandrasekhar

I cannot clearly say anything that will relate to Ramanujan as a Mathematician, particularly in this company which includes, among others, Professors Richard Askey, Bruce Berndt, and George Andrews, who have devoted years to exploring and following his many trails. But I do share with Ramanujan the same cultural background in our early formative years: both of us originate in a common social background -- he from Kumbakonam and I from Tanjore, both ancient centers of Tamil culture and not very far apart. Besides, Ramanujan's parents and my own grandparents lived in very similar social and financial circumstances. On this account I can probably visualize Ramanujan's background better than even my younger Indian colleagues of later generations.

With this common background, I can perhaps throw some light on some conflicting statements that have been made about Ramanujan and 'God' by some of his Indian contemporaries. I refer here particularly to the colorful stories concerning Ramanujan's devotion to the Namakkal Goddess.

Quite generally, it may be stated that among those who were brought up in south India during the first two decades of this century, there was (and probably still is) very little correlation between observance and belief. In particular, I can vouch from my own personal experience that some of the

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[2] *Ramanujan Revisited*, edited by G.E. Andrews R. Askey, B.C. Berndt, R.A. Rankin and K.G. Ramanathan, Academic Press, 1988.

[3] Remarks made at the Banquet on June 3, 1987

'observances' that one followed were largely for the purposes of not offending the sensibilities of one's parents, relations, and friends.

I can say a good deal on these matters, but I shall only state that I do not accept what has commonly been said and written about Ramanujan's religious beliefs. I corresponded with Hardy on this matter while he was preparing for his Harvard Lectures; and I am personally much more inclined to accept his view as expressed in a letter to me dated February 19, 1936.

> ... And my own view is that, at bottom and to a first approximation, R. was (intellectually) as sound an infidel as Bertrand Russell or Littlewood....
>
> One thing I am sure. R was not in the least the 'inspired idiot' that some people seem to have thought him. On the contrary, he was (except for a period when his mental equilibrium was definitely upset by illness) a very shrewd and sensible person: very individual, of course, and with a reasonable allowance of the minor eccentricities of genius, but fundamentally normal and sane.

And this view of Hardy's is corroborated by K. Ananda Rao, himself a mathematician of distinction, who had been Hardy's student and Ramanujan's contemporary in Cambridge. Ananda Rao is well known and remembered for his contributions to the theory of Tauberian theorems, function-theory and the theory of Dirichlet series. He has written:

> In his nature he was simple, entirely free from affectation, with no trace whatever of his being self-conscious of his abilities. He was quite sociable, very polite and considerate to others. He was a man full of humour and a good conversationalist, and it was always interesting to listen to him. On occasions when I met him, we used to talk in homely Tamil. He could talk on many things besides mathematics ....

This view of Ananda Rao is not surprisingly the same as Hardy's. He has written,

> ... the picture which I want to present to you is that of a man who had his peculiarities like other distinguished

men, but a man in whose society one could take pleasure, with whom one could drink tea and discuss politics or mathematics.

Let me now turn to the role of Ramanujan in the development of science in India during the early years of this century.

Perhaps the best way I can give you a feeling for what Ramanujan meant to the young men going to schools and colleges during the period 1915-1930 is to recall for you the way in which I first learned of Ramanujan's name.

It must have been a day in April 1920, when I was not quite ten years old, when my mother told me of an item in the newspaper of the day that a famous Indian mathematician, Ramanujan by name, had died the preceding day; and she told me further that Ramanujan had gone to England some years earlier, had collaborated with some famous English mathematicians, and that he had returned only very recently, and was well known internationally for what he had achieved. Though I had no idea at that time of what kind of a mathematician Ramanujan was, or indeed what scientific achievement meant, I can still recall the gladness I felt at the assurance that one brought up under circumstances similar to my own, could have achieved what I could not grasp. I am sure that others were equally gladdened. I hope that it is not hard for you to imagine what the example of Ramanujan could have provided for young men and women of those times, beginning to look at the world with increasingly different perceptions.

The fact that Ramanujan's early years were spent in a scientifically sterile atmosphere, that his life in India was not without hardships, that under circumstances that appeared to most Indians as nothing short of miraculous, he had gone to Cambridge, supported by eminent mathematicians, and had returned to India with every assurance that he would be considered, in time, as one of the most original mathematicians of the century -- these facts were enough -- more than enough -- for aspiring young Indian students to break their bonds of intellectual confinement and perhaps soar the way that Ramanujan had.

It may be argued, perhaps with some justice, that this was

a sentimental attitude: Ramanujan represents so extreme a fluctuation from the norm that his being born an Indian must be considered to a large extent as accidental. But to the Indians of the time, Ramanujan was not unique in the way we think of him today. He was one of others who had, during that same period, achieved, in their judgement, comparably in science and in other areas of human activity. Gandhi, Motilal and Jawaharlal Nehru, Rabindranath Tagore, J.C. Bose, C.V. Raman, M.N. Saha, S.N. Bose, and a host of others, were in the forefront of the fermenting Indian scene. The twenties and the thirties were a period when young Indians were inspired for achievement and accomplishment by these men who they saw among them.

I do not wish to leave the impression that Ramanujan's influence was only in this very generalized sense. I think it is fair to say that almost all the mathematicians who reached distinction during the three or four decades following Ramanujan were directly or indirectly inspired by his example.

But Ramanujan's name inspired not only ambitious young men planning scientific careers; it also stimulated to action those with public concern. Let me give one example.

When I was a student in Madras one of my classmates (who came from a very wealthy family) was one Alagappa Chettiar. We became good friends; but our lives diverged along different paths after 1930. In the years before and during the second world war, Alagappa Chettiar prospered as an entrepreneur and became a noted philanthropist. He was in fact knighted by the British government.

During the late forties after the war, Sir Alagappa Chettiar (as he was then) wrote to me inquiring if it might be useful for him to found a mathematical institute in Madras named after Ramanujan. I enthusiastically supported the idea; and when I returned to India briefly in 1951, the Ramanujan Institute had been founded a few months earlier. Its first director, T. Vijayaraghavan, was one of the most talented among Hardy's former students; he died at a comparatively early age in 1955. C.T. Rajagopal, a student of Ananda Rao, took over the directorship from him. Already at that time the financial

status of the Institute seemed shaky, since Alagappa Chettiar's fortune was melting away.

In April 1957, when Alagappa Chettiar died, the fate of the Institute hung in the balance; Rajagopal wrote to me that the Institute 'will cease to exist on the first of next month,' whereupon I wrote to the Prime Minister (Jawaharlal Nehru), explaining the origin of the Institute and the seriousness of its condition. Nehru's prompt answer was refreshing: 'Even if you had not put in your strong recommendation in favour of the Ramanujan Institute of Mathematics, I would not have liked anything to happen which put an end to it. Now that you have also written to me on this subject, I shall keep in touch with this matter and I think I can assure you that the Institute will be carried on.' And it was; but haltingly and precariously for the next twelve years. It is at this Institute in Madras that Ramanujan's Centennial will be celebrated by an International Conference in December.

There is very little more I can say. My own view, sixty-six years after my first knowing of his name, is that India and the Indian scientific community were exceptionally fortunate in having before them the example of Ramanujan. It is hopeless to try to emulate him. But he was there even as the Everest is there.

On Ramanujan Institute, founded by Alagappa Chettiar, in 1951, which was absorbed in 1957 by the University of Madras to become the Center for Advanced Study in Mathematics and which moved into a modern building[4] in 1972, Kanigel wrote [X]: "... it does not specialize in areas of mathematics Ramanujan pursued, nor does its name bear anything like the luster of its namesake". This ought to make us sit up, having left it to Andrews, Askey and Berndt to make Ramanujan and his mathematics vibrant with life even today.

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[4]This is situated on the Wallajah Road, on the other side of which is the famous Chepauk Cricket Ground. As stated earlier, in another footnote, attempts to get the street named after Ramanujan met with no success till date.

Ramanujan's Photograph

The one authentic photograph of Ramanujan which was first published in the book on *Ramanujan:Twelve Lectures on subjects suggested by his life and work*, by Hardy [IV], was taken by Ramanujan when he showed signs of imporvement in his health after prolonged treatment in several nursing homes and sanatoria. This photograph reproduced at the beginning of this book was taken, for the passport, in Feb. 1919.

The letters of Dr. Chandrasekhar to Hardy on the Photograph of Ramanujan, written on August 4 and December 28, 1937, are reproduced below:

The University of Chicago

Yerkes Observatory

WILLIAMS BAY, WIS

1937 August 4

Dear Hardy,

I remember your telling me that when Ramanujan's collected papers were being edited, it was your original intention to include a portrait of R., but eventually you had to abandon the plan as no good photograph was available.

So, when I was in India last summer, I made an effort to find a reasonably good photograph. I met Mrs. Ramanujan – his wife, who incidentally is having a rather difficult life, some of her unscrupulous relatives having swindled her out of such financial resources as R. had left her – and it transpired that the only photograph of R (other than the one where he is "cap and gown") available is his pass-port photograph taken prior to his leaving England.

I had a negative taken of his passport photograph (which is now in my possession) and had an enlargement professionally made. I am sending you by separate cover an enlarged photograph of R, and as you will see it is a reasonably

good one. I am not in a position to judge how 'true' it is, but I have it on Mrs.Ramanujam's authority that the rather worried look R has in the picture was extremely frequent during his last year.

I do not know if you would want to include this photograph in your book on R – I refer to your Harvard lectures and I do not know either if the enclosed enlargement would be good enough for reproduction purposes. In any case I think a good professional must be able to use the original negative I have and if it should be needed, I shall be only too glad to loan it. I have some more spare copies of the enlargements of R and if you should know others who may like to have a copy of R's photograph perhaps you could ask them to write to me.

With kindest regards,
Yours very sincerly,
Chandrasekhar.

P.S. In any case, I should be glad to know what you think of the photograph.

Some of the copies of the photograph referred to in this letter of Chandra to Hardy are still in the collection of papers of Ramanujan at the Wren Library of the Trinity College, Cambridge.

The University of Chicago
Yerkes Observatory
WILLIAMS BAY, WIS
1937 December 28

Dear Hardy,

Many thanks for your letter of Dec. 15. I am very glad to learn of your intention to include the photograph in your book.

I am enclosing the negative from which the enlargement I sent you was made. Also a rather good copy of the same size as the negative and four rather poor copies of the enlargement. I am afraid that my own efforts at copying from the enlargement proved less successful than I had hoped! I am however certain that a professional photographer can make even better enlargements from the negative than the one I sent you.

My impression is that the college has no portrait of R. in its collection of fellows – perhaps you would be so kind as to pass on one of the four copies I am sending.

With the seasons greetings and with kind regards

Yours very sincerely,

S. Chandrasekhar.

Jawaharlal Nehru, the first Prime Minister of Independent India, reputed for his book *Discovery of India*, was genuinely interested in Science and Technology. We have already seen that Chandrasekhar appealed to him for the continuance of the Ramanujan Institute. The letter of Chandrasekhar to Nehru and the latter's reply are preserved in the Nehru Memorial Museum[5].

We reproduce here the reference to Ramanujan contained in Nehru's *Discovery of India*:

Mathematics in India inevitably makes one think of one extraordinary figure of recent times. This was Srinivasa Ramanujam. Born in a poor Brahmin family in South India, having no opportunities for a proper education, he became a clerk in the Madras Port Trust. But he was bubbling over with some irrepressible quality of instinctive genius and played about with numbers and equations in his spare time. By a lucky chance he attracted the attention of a mathematician who sent some of his amateur work to Cambridge in England. People there were impressed and a scholarship was

[5]Our efforts to get copies of these letters for inclusion in the Indian edition of the book on *Ramanujan: Letters and Commentary* by Bruce C. Berndt and Robert A. Rankin [XIV] did not meet with success.

arranged for him. So he left his clerk's job and went to Cambridge and during a very brief period there did work of profound value and amazing originality. The Royal Society of England went rather out of their way and made him a Fellow, but he died two years later, probably of tuberculosis, at the age of 33. Professor Julian Huxley has, I believe, referred to him somewhere as the greatest mathematician of the century.

Ramanujam's brief life and death are symbolic of conditions in India. Of our millions how few get any education at all, how many live on the verge of starvation; of even those who get some education and have nothing to look forward to but a clerkship in some office on a pay that is usually far less than the unemployment dole in England. If life opened its gates to them and offered them food and healthy conditions of living and education and opportunities of growth, how many among these millions would be eminent scientists, educationists, technicians, industrialists, writers and artists, helping to build a new India and new world?

A remarkable story

The Srinivasa Ramanujan Medal of the Indian National Science Academy was conferred on Dr. Chandrasekhar, in 1968. So, that year, Chandra came to India and gave the Nehru Memorial Lecture and the Ramanujan Lecture. Besides he had a meeting with Mrs. Ramanujan. What happened on that occasion is recorded to us in the chapter on *Conversations with Chandra* by Kameshwar Wali in his book on Chandra [67, p. 262], which is interesting reading indeed and so it is reproduced below:

I learned of the incident from a conversation I had with Hardy at dinner in Trinity during the spring of 1936. Hardy arrived a little late but bandaged, and he sat at the table opposite me. The fact that he was bandaged naturally aroused the concern of those around him, and when questioned, he told a remarkable story. It's rather elaborate. You may want to reproduce it from the talk I gave to a private club at the University of Chicago.

[The following is from S. Chandrasekhar, Notes and Records of the Royal Society 30 (1976): 249.[6]]

It appeared that he [Hardy] was in London during the day and while crossing Piccadilly Circus a motorcycle hit him and dragged him along. He was bruised but only superficially. Nevertheless, Hardy was escorted to the Scotland Yard by the policeman who had arrested the cyclist in order that Hardy could report what had happened. After Hardy had given the appropriate evidence and was about to leave, a messenger came to him and told him that a senior officer of the Scotland Yard wanted to see him. Hardy was slightly surprised. But the officer treated him with great courtesy, asked him to be seated and said, "Professor Hardy, I have been wanting to see you for many years. In fact, I have waited for this occasion for seventeen years. Do you know that we have had evidence in our files here to arrest you for giving false evidence?" Hardy was a little surprised and the officer continued. "Do you remember, Professor Hardy, that in February 1918 an Indian mathematician had tried to commit suicide by falling before the train in an underground tube station? His intention was thwarted and he was arrested and brought to Scotland Yard. I was in charge of that case. And you arrived later to give evidence." At this point I should digress, even as Hardy did while narrating his encounter with the officer of the Scotland Yard.

Ramanujan had been quite ill during the winter of 1917 and apparently in a state of extreme depression. Ramanujan in fact had tried to commit suicide in the manner described by the officer; but by a series of miracles (like the switch being turned off by a guardsman, and the train coming to a stop just a few feet ahead of where Ramanujan had fallen) he had been saved. To continue with the story.

"When you arrived at Scotland Yard, Professor Hardy, you told us that Ramanujan was a Fellow of the Royal Society and as such could not be arrested. We released Ramanujan and you left apparently believing that you had bluffed us; but in fact you had not. You knew as well as we did that Fellows of the Royal Society are not immune from arrest. But you also told a lie. At the time of this incident Ramanujan was not in fact a Fellow of the Royal Society. But you knew that he would be elected a month later but that is not the same thing. Nevertheless, on inquiry we found that the man whom we had arrested was indeed reputed to be a

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[6]This account is now in print in *Littlewood's Miscellany*, ed. Bela Bollobas (London: Cambridge University Press, 1986). Also, the Madras city based newspaper *The Hindu* carried the story, on 17th Nov. 1968, under the heading: *When U.K. Policemen winked at an offence.*

great mathematician and we in Scotland Yard did not want to spoil that life. And so we let you believe that you had convinced us. But I have always hoped that an occasion would arise when I could tell you that we knew all along that you were telling a falsehood and had perjured yourself. The occasion has now come; but I am not going to arrest you."

I cannot quite recall whether it was on this same occasion, or on a later occasion, that Littlewood referred to Ramanujan's attempt to commit suicide in connection with his (Ramanujan's) election to a Fellowship at Trinity. There is apparently a rule that one who is medically insane cannot be elected to a Fellowship at Trinity. And Littlewood (who was one of the electors during the year that Ramanujan was a candidate) was afraid that Ramanujan's "insanity" in having attempted suicide might be brought up to disqualify his election; and on that account Littlewood said that he had gone to the electors meeting with a medical certificate to the effect that Ramanujan was not afflicted by insanity. And Littlewood added that he was extremely glad that no occasion arose which required him to produce the certificate.

[Our conversation continues.]

You know there is a sequel to that story. In 1968 I gave the Nehru Memorial Lecture and the Ramanujan Lecture at the Indian National Academy of Sciences. They now call it the Indian National Science Academy. And, since it was the Ramanujan Memorial Lecture, I told the stories which Hardy and Littlewood had told me about Ramanujan's election to Trinity Fellowship and the Fellowship of the Royal Society. In that context, I mentioned Ramanujan's attempted suicide. But it appeared in the newspaper the following day with big headlines, "Ramanujan tried to commit suicide," and told the story in a garbled way. I was astonished at the opposition it created. Everybody thought I was ill-advised in telling that story. I went to Bangalore to see Raman, and one of the first things he said was, "Why do you want to defame that man?" Then there was a letter in the Times of India *saying that my motive in recounting the incident was to enhance my own personal reputation at the expense of Ramanujan's. I was very depressed by that reaction. A few days later when I went to*

Madras, I was met at the library by a young man who introduced himself as a neighbor of Mrs. Ramanujan; and he told me that Mrs. Ramanujan was anxious to see me. I was naturally worried that she would be very upset about it. However, she told me that my account of Ramanujan's attempted suicide had cleared up certain things about her life, which was so tragic. I have written up the things she told me in the Royal Society notes, but not for the public, because I don't know whether it is the kind of thing one should write without giving proper account of the whole story. Just one remark, Mrs. Ramanujan told me, for instance, that Ramanujan's mother put every kind of obstacle in their married life. ...

It is known (recorded, for example, by Hardy) that during Ramanujan's years in England, he received very few letters from his family in India. And Hardy attributed Ramanujan's depression during his later years in Cambridge to his "misunderstandings" with his family. Ramanujan used to write regularly to his wife, but apparently she never saw any of those letters since his mother apparently did not pass them on. Ramanujan's wife could not write to him because she did not have the money for postage. ...

It throws an entirely different light. Yet he made some of his great contributions to mathematics during his four years in England. So I did write of my visit with Mrs. Ramanujan and sent it to the Royal Society Archives in London with his photograph and other things. They asked me whether they should publish it in their notes and records. I have not given them permission, but I have a feeling that it should be published sometime.

Today, Mrs. Ramanujan is no more. The time is ripe for the Royal Society to publish the above mentioned write-up of Chandra on his visit to meet Mrs. Ramanujan in 1968 and other documents which Chandra left with the Royal Society in its Notes and Records section[7].

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[7]Professors Bruce C. Berndt and Robert A. Rankin did look into these Notes left behind by Chandra, before they completed their work on *Ramanujan: Letters and Commentary* [XII] and I understand that these Notes are details of what we already know (private communication from Prof. Berndt)

6. Books and Busts

There are two aspects of interest to biographers and mathematicians regarding Ramanujan: his life and his work. Mathematicians, who are interested in his work, have to contend with not only his publications in journals which are precise and profound, but also with his Notebooks[1] which are a treasure house of intriguing results stated without proofs and lacking perspective with contemporary mathematical work. Those who attempt to write biographies or articles on Ramanujan have to surmount the time barrier to reconstruct a story from all the indirect information accessible and to them, as stated earlier, Hardy on Ramanujan [IV] is like Boswell on Samuel Johnson. The challenge to the mathematicians who work on any of his thousands of recorded results, which are still shrouded in mystery, is to prove the same with what was accessible to Ramanujan in those days in the form of books and publications. While the individual writer's perception of Ramanujan will depend upon his/her background and imagination, the task of the mathematician is perhaps unenviable, in comparison.

Anyone who ever heard of Srinivasa Ramanujan and reads the compelling rags-to-intellectual-riches story of Ramanujan contained in the two Notices, one by G.H. Hardy and the other by Dewan Bahadur R. Ramachandra Rao and P.V. Seshu Iyer, published in the *Collected papers of Srinivasa Ramanujan* [III], would be moved by the achievements of the unorthodox mathematical genius under adverse circumstances. The lack of formal education, lack of appreciation and a job, in the beginning of his career and ill health during the last few years of his life, did not prevent him from being creative in Mathematics. This is indeed something not easy to comprehend, for often one would buckle under similar trying circumstances.

The aforesaid Notices are the earliest biographical sketches of the life of Ramanujan in books. An authentic version of not only the life but also a critical appraisal of the work of Ramanujan is contained in the most valuable contribution of Hardy, his book *Ramanujan: Twelve Lectures on subjects suggested by his life and work* [IV], from which we have culled extensively information for the earlier chapters of this vignette.

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[1] *'...that chaotic collection of thousands of formulas'* (G.E. Andrews [71], p.87).

After the 75th Anniversary (in 1962), the Mathematics Teacher, P.K. Srinivasan, of Muthialpet High School, Madras, put in a supreme effort to bring out a two part Memorial publication: *Ramanujan Memorial Number, Vol.1: Letters and Reminiscences* [I] and *Vol.2: An inspiration* [II]. At about the same time, the renowned Library Scientist, S.R. Ranganathan, who had contact with Hardy in the 1920s, brought out a biography entitled *Ramanujan: the man and the mathematician* (Asia Publishing House, 1967).

The centenary year - Ramanujan Revisited

In the birth centenary year of Ramanujan, celebrated all over the world, in 1987, there were several meetings whose theme was the work of Ramanujan and its continuing influence and impact on the growth and development of Mathematics. The Annamalai University and the Anna University had conferences preceding an International Conference organized by the National Board for Higher Mathematics at the Institute of Mathematical Sciences in Madras, from Dec. 22 to Dec. 27, 1987, which was inaugurated by the then Prime Minister Rajiv Gandhi. In this meeting, Professors G.E. Andrews, R. Askey, B.C. Berndt, S. Chandrasekhar, R.A. Rankin, A. Selberg, D. Zagier and others from abroad, and Professors R.P. Agarwal, S. Bhargava, S. Raghavan, S.S. Rangachari, C.S. Seshadri, A. Verma and several others from India took part. Unfortunately the death of the Chief Minister of Tamil Nadu, M.G. Ramachandran, on the 24th Dec., threw all the arrangements out of gear for two days, until the burial took place, in what could be called as a characteristic behaviour of the people of a country prone to excesses even on such solemn occasions.

Immediately after the Madras meeting, an International Conference on Number Theory was held at Kandy, Sri Lanka, hosted jointly by the Institute of Mathematical Sciences, Madras, and the Institute of Fundamental Studies, Kandy, in which Professors A. Selberg, R.A. Rankin, B.C. Berndt, E.C.G. Sudarshan, D. Zagier, G. Bhamathi, the author and others participated. The author had the responsibility of organizing this meeting and coordinating it with the Institute in Kandy.

The Madras meeting was significant for it was at this meeting that the 'Lost'

notebook was released by the then Prime Minister of India who gave the first copy of this publication brought out by Narosa Publishing House to Mrs. Ramanujan. We give below a brief outline of what this significant publication contains:

The 'Lost' notebook of Ramanujan, discovered by Prof. George E. Andrews, in the spring of 1976, had been earlier 'found' in November 1965 by Dr. J.M. Whittaker lying loose on the floor among Watson's papers. These papers were obviously part of papers dispatched from India to Hardy, in August 1923, a few years after the death of Ramanujan. It is not clear after all these years whether these were scrutinized or not by Watson and Wilson who took up the task of editing the notebooks at the instance of Hardy. However, Prof. Robert A. Rankin has classified all the papers which are now in the Wren library and this work of classifying was done in the year 1968. Today, these papers are found in one of three large card board boxes (each foolscap size box is about 2 inches thick) in the archives of the Wren library. One of the items (subsequently catalogued and) accessible with Add.Ms.a.94 is:

S.Ramanujan: "Lost Notebook" on q-series and similar types, n.d.

In the appendices at the end of this book the reader will find all the papers of Ramanujan with the Wren library, the National Archives in New Delhi and the Tamil Nadu Archives in Chennai.

Srinivasa Ramanujan – The Lost Notebook and Other Unpublished Papers [101], has an introduction by George E. Andrews. The following Publisher's Note is self-explanatory as regards the contents of this book:

> *In this collection of unpublished manuscripts of Srinivasa Ramanujan, we have first reproduced a major portion of the "Lost Notebook", consisting of 90 unpaginated sheets representing his work on q-series and other topics. The "Lost Notebook" was brought to light in the spring of 1976 as "part of the Watson bequest" by Professor G.E. Andrews whose introduction precedes the text of the "Lost Notebook" reproduced here. This is followed by letters written by Ramanujan to G.H. Hardy during his stay in English nursing homes, on many mathematical topics including "coefficients in the $1/g_3$, and $1/g_2$ problems" as well as the only available remnant of his famous letter dated 12 January 1920 on mock theta*

functions. Various sheets in Ramanujan's handwriting seemingly related to these letters have been inserted close to them.

Next, we have provided a hitherto unpublished manuscript of Ramanujan's on "Properties of $p(n)$ and $\tau(n)\cdots$" dealing with congruence relations satisfied by these arithmetic functions. One may find, thereafter, 28 sheets copied from the "Loose Papers" of Ramanujan held in the Trinity College Library; these include a note on "Reciprocal functions", another concerning "Approximate summation of series involving prime numbers", Ramanujan's marvelous discoveries on Euler products of Dirichlet series associated to modular forms and his famous "forty identities", with relevant sheets in Ramanujan's handwriting juxtaposed. ...

The subsequent 117 pages include Ramanujan's unpublished work related to various papers of his, especially in continuation of the one entitled "Highly composite numbers" and "On certain trigonometrical sums ... with Hardy's noting there on; besides, one may find here class invariants listed by Ramanujan and a host of interesting identities of an arithmetic nature.

At the end, we have taken care not to miss out on interesting letters from J.E. Littlewood to G.H. Hardy, G.H. Hardy to Ramanujan, G.H. Hardy to G.N. Watson, etc. with a bearing on Ramanujan's work and various other letters of significance. Fragments in Ramanujan's handwriting have all been arranged as appropriately as possible. This last section contains also E.H. Neville's letter on Ramanujan extracted from Nature of 20 January 1921.

From the above Publisher's Note it is clear that the volume brought out on the occasion of the Ramanujan birth centenary as a companion volume to the two volume facsimile edition of Ramanujan's notebooks [VII] contains a lot more information which would be of interest to the mathematicians. Narosa Publishing House has thus the set of three volumes on the notebooks of Ramanujan which is a must for any good mathematics library anywhere in the world.

The following are some of the other significant meetings throughout the year held at many centers in India and in other countries too:

- On Dec. 22, 1986, year long celebrations at the Ramanujan Institute for Advanced Study in Mathematics of the University of Madras, were inaugurated by Prof. R.P. Agarwal, the then Vice Chancellor of Rajasthan University.

- Prof. R.P. Agarwal conducted a three week National Instructional Seminar on *Ramanujan and Special Functions*, from May 15 – Jan. 6, 1987, at Mt. Abu, Rajasthan. This Seminar funded by the National Board for Higher Mathematics (NBHM), India, was inaugurated by Prof. M.S. Raghunathan, the then Chairman of the NBHM.

- About 125 Mathematicians form the world over gathered at the University of Illinois, Urbana-Champaign to celebrate the centenary of Ramanujan's birth, due to the initiative of Prof. Bruce C. Berndt.

- A National Instructional Conference on Ramanujan's Circle Method, sponsored by the Central University at Pondicherry and the NBHM, was held at Pondicherry from Sept. 21 – Oct. 5, 1987.

- A *Srinivasa Ramanujan Centenary Seminar on Algebra and its Applications*, was held at the Ramanujan Institute for Advanced Study in Mathematics of the University of Madras, from Oct. 26 – 312, 1987, as part of its year long celebrations of the centenary.

- The Association of Mathematics Teachers of India (AMTI) held its XXII Annual Conference as a National homage to Ramanujan, at Kumbakonam Town Higher Secondary School, Kumbakonam, where Ramanujan had studied. A Hall in this school is named after its famous student as *Ramanujam Hall* on this occasion.

- The Ramanujan Mathematical Society, which came into existence due to the initiative of Prof. R. Balakrishnan and others, in 1985, conducted a Ramanujan Centennial Conference at Annamalai University, Annamalai Nagar, from Dec. 15 – 18, 1987.

- An *International Conference on Mathematics* was organized by the Anna University, Madras, in collaboration with the Tamil Nadu Academy of Sciences, Third World Academy of sciences and Committee for Science and Technology Development, at the University's Campus in Guindy, from Dec, 19 – 21, 1987. The Computer Center of this University has been named after Ramanujan.

- The *Ramanujan Birth Centenary Year International Symposium on Analysis* was organized by the Department of Mathematics of the University of Pune, from Dec. 26 – 28, 1987.

- The Indian Mathematical Society's 53rd Annual Conference was devoted to the celebration of the Ramanujan Birth Centenary and was held at the University of Gorakhpur, Gorakhpur, from De. 30, 1987 – Jan. 2, 1988.

- Several institutions had one-day symposia to mark the centenary:

  – Bharathiar University, Coimbatore, on Aug. 12, 1987.

  – The Indian Institute of science, Bangalore on Sept. 16, 1987.

  – the S.N. Bose School for Advanced Study and research in Mathematics and Mathematical Physics, Calcutta, on Dec. 31, 1987.

As a consequence several conference proceedings, published in 1987 – 88, are today available covering a whole gamut of work suggested or inspired by the work of Ramanujan. Most notable amongst these is: *Ramanujan Revisited* [XVI], edited by George E. Andrews, Richard A. Askey, Bruce C. Berndt, K.G. Ramanathan and Robert A. Rankin. It is on this occasion that Dr. S. Chandrasekhar gave a Banquet address *On Ramanujan*, on June 3, 1987, which we have reproduced in chapter 5 of this book. In another fascinating article in this Proceedings, Freeman J. Dyson, the reputed Physicist, makes revealing statements about the natures of G.N. Watson and George E. Andrews, in his article entitled: *A walk through Ramanujan's garden*:

> *I am grateful to the orgnizers of this conference, and to Bruce Berndt and George Andrews in particular, for bringing me here and giving me a chance to enjoy all the new flowers that have been growing in recent years in Ramanujan's garden. George Andrews is now the chief gardener and is doing a magnificient job. He is,*

if I may say so, doing a better job than George Watson who used to be chief gardener in the old days. Watson was chief gardener in the 1930's and worked hard to develop and elucidate Ramanujan's ideas, especially the theories of singular modulii and mock theta-functions[2]*. But Watson was the sort of gardener who liked to keep human visitors as far as possible away from the flowers. He stood at the entrance to his garden like the goddess Proserpine. Proserpine stands at the entrance to her garden in the poem of Swinburne which Watson quoted at the end of his famous lecture*[3] *on the mock-theta functions:*

> *"Pale, beyond porch and portal,*
> *Crowned with calm leaves, she stands*
> *Who gathers all things mortal*
> *With cold immortal hands."*

Watson was like that. My good friend Oliver Atkin had the misfortune to work on Ramanujan's discoveries during the years when Watson was in charge of the garden. Atkin discovered and proved many new congruence properties of partitions, extending and developing Watson's own work[4]*. Atkin wrote letters to Watson, keeping him abreast of the progress and asking him for help and advice. When Watson died in 1965, Atkin wrote to me, "I expect you saw that Watson died. My correspondence with him is thus closed. Final score of letters: twenty-nil."*

George Andrews is different. He does not stand like Proserpine, gathering all things mortal with cold immortal hands. He answers letters. He even likes to have live human beings in his garden, trampling over the flower-beds. He gives help and encouragement to anyone who comes visiting, whether it is an elderly professor like me or a raw student. As a result, the garden is now blooming and sprouting more vigourously than ever. George Andres also

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[2]J.M. Whittaker, *George Neville Watson*, Biographical Memoirs of Fellows of the Royal Society, **12** (1966) 512-530.

[3]G.N. Watson, *The Final Problem: an account of the mock theta functions*, J. London Math. Soc. **11** (1936) 55-80.

[4]For example, A.O.L. Atkin, *Proof of a Conjecture of Ramanujan*, Glasgow Math. Jour. **8** (1967) 14-32.

enlarged the territory of the garden by finding the famous lost notebook of Ramanujan, the notebook which Watson certainly had in his possession for thirty years but apparently never examined. Andrews not only examined the notebook for the first time but also explained and displayed for all to see the treasures that it contains[5]

Another noteworthy publication is the Proceedings of the International Centenary Conference, held at the Anna University, Madras, from Dec. 19 – 21, 1987, edited by Krishnaswami Alladi [97] and brought out as a part of the Lecture Notes in Mathematics (No. 1395) series of Springer-Verlag (1989). That same year, 1987, a biography [98] of Ramanujan by the Tamil writer T.V. Rangaswami, under his pseudonym Ragami, appeared as a serial in the weekly edition of the popular Tamil newspaper Dinamani. Subsequently these articles, thirty-one in number, were strung together in the form of a book by that author. A biography by Dr. Jasjit Singh, a popular writer, also appeared in a series of four articles [99] in the Mathematics Education journal sponsored by the University Grants Commission. While these were meant for the general readers, two special publications by Madurai Kamaraj University deserve to be mentioned here. One of these is *Srinivasa Ramanujan (1887 - 1920): A Tribute* [IX], with expository articles by the present author, Professors G.E. Andrews, R. Askey, B.C. Berndt, R.P. Agarwal, A. Verma, S. Bhargava, M.V. Subbarao and T. Soundararajan. Another is a Technical Report on the *Development of Elliptic Functions according to Ramanujan* by Dr. K. Venkatachaliengar [XIV].

Earlier in this work we have extensively referred to the books: *Collected Papers of Srinivasa Ramanujan* [III]; *Ramanujan: Twelve Lectures on subjects suggested by his life and work* [IV] and the books containing proofs and perspectives for everyone of the more than 3000 results, noted down by Ramanujan during 1903 – 1914 in his Notebooks, by Bruce C. Berndt in *Ramanujan Notebooks*, Parts I to V [VIII]. In this section, brief reviews of some of the other books on Ramanujan are presented. The selection is based on the author's familiarity and predelictions.

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[5]G.E. Andrews, *An Introduction to Ramanujan's 'Lost' Notebook*, Am. Math. Monthly **86** (1979) 89-108, and a series of papers in "Advances in Mathematics" [56], (1981-1987).

S.R. Ranganathan's biography (1967)

In the Introduction to this book (of about 130 pages), S.R. Ranganathan, the renowned Librarian of the Madras University, who introduced the colon system – for classification of books and periodicals – in Library Science, reveals his role [VI, p.9] in the first biographical notices on Ramanujan, which appeared in the Collected Papers [III]:

> *In 1923, it was decided that a biography of Ramanujan should be given at the beginning of his Collected Papers. The University of Madras appointed a Committee to write the biography. It consisted of E.M. MacPhail, the Vice-Chancellor, P.V. Seshu Ayyar, Professor of Applied Mathematics in the Presidency College and Secretary of the Indian Mathematical Society, and R. Ramachandra Rao, Education Secretary of the Government of Madras and former President of the Society. As a junior member of the staff of the Department of Mathematics of the Presidency College, it fell to my share to prepare a draft of the biography. As Approved by the Committee, it was published in the Collected Papers in 1927.*

The inaccessibility of this biography in the Collected Papers and the ignorance about Ramanujan and his career among University graduates, *who had not even heard of his name* and whose number *had increased considerably* between 1950 and 1960, were the motivations[6] for S.R. Ranganathan to write

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[6]My own motivation is not very different: I was surprised to find, in 1987, the centenary year of Ramanujan's birth, in Kumbakonam, in the street in which Ramanujan lived, when I asked a couple of shop keepers and some school children and younger people about the house where Srinivasa Ramanujan lived, total unawareness. Some asked *where he worked* or *where did he retire from?* Eventually, an aged, orthodox, Sri Vaishanavite, pointed out the house to me. Even now, even among College and University teachers, all that is known about Ramanujan amounts to the bare facts that he was uneducated, poor and he became famous after he came into contact with an English mathematician. This work is being written with the hope that it (would be low priced and) will be easily accessible in all Schools and Colleges and act as a Pandora's box on Ramanujan and his eternal mathematics, to students and lovers of mathematics.
I cannot resist drawing the attention of the reader to a different field and a different context, in which I admired the awareness of the public and tourists to Beethoven's *Geburt Haus* (house where he was born) in Bonn and the efforts made by the Germans to preserve their heritage. On the occasion of Beethoven's 150th Birth Anniversary, when I was in Bonn, in 1984, the Mayor of that city announced that the whole city would pay its homage to

this brief biography entitled: *Ramanujan the man and the mathematician* [VI]. An All India Radio, Madras, talk on Ramanujan, by him, on the occasion of the 75th (Platinum Jubilee) Birthday of Ramanujan, in 1962, and the response – *letters from a couple of the surviving friends of Ramanujan* – to that talk, spurred Ranganathan to get reminiscences from *all of them*, including Mrs. Ramanujan, over a period of two years, to embark on the task of writing a biography concerning the human aspects of Ramanujan.

Ranganathan points out that *the Contents Page forms a clear Conspectus of the book* and so we give here the table of Contents [IV]:

A Introduction
B Ramanujan: A Puzzle
C Super-Activity Period 1: 1907 to 1911
D Economic Sufficiency
E University Setting
F Ramanujan's Life Abroad
G Super-Activity Period 2: 1914 to 1918
H Precipitation of Honours
J Ramanujan's Health Fails
K Memorial
L Ramanujan's Notebooks
M Reminiscences of Friends
N Ramanujan the Man
P Ramanujan the Mathematician
Q Works By and On Ramanujan
R Up-to-date Bibliography
S Bio-data
Index

An anecdote in this book reveals the simplicity of Ramanujan to mundane details, as told by the renowned Indian Statistician Dr.P.C. Mahalanobis, F.R.S., who was a student of King's College, Cambridge (from October 1913):

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the great Maestro by broadcasting not only on the Radio but also in the Market Square his Ninth Symphony, with the citizens tuning their Radios and opening their windows to fill the air with his music to reach the Heaven!

I was fortunate in forming a good friendship with Ramanujan very soon. It came about in a somewhat strange way. One day, soon after his arrival, I went to see Ramanujan in his room in Trinity College. It had turned quite cold. Ramanujan was sitting very near the fire. I asked him whether he was quite warm at night. He said that he was feeling the cold though he was sleeping with his overcoat on and was also wrapping himself up in a shawl. I went to his bedroom to see whether he had enough blankets. I found that his bed had a number of blankets but all tucked in tightly, with a bed cover spread over them. He did not know that he should turn back the blankets and get into the bed. The bed cover was loose; he was sleeping under that linen cover with his overcoat and shawl. I showed him how to get under the blankets. He was extremely touched. I believe this was the reason why he was so kind.

It is an interesting fact that after Ranganathan left the University of Madras, he became the Chief Librarian at the Benares Hindu University. It was he who drew the attention of Dr. K.S. Krishnan (the student and collaborator of Sir C.V. Raman) regarding the notebooks of Ramanujan. At that time, Dr. K.S. Krishnan was a neighbour of Pundit Jawaharlal Nehru in Allahabad and he induced Ranganthan to write to Dr. Homi. J. Bhabha at the Tata Institute ofFundamental Resarch in Bombay. Subsequently, Ranganathan, Krishnan and Bhabha called on Pundit Nehru and as a consequnce the facsimile edition of the notebooks of Ramanujan [VII] were published without commentary (described in chapter 3).

Review of Kanigel's book[7] (1991)

In late 1987, when magazines and newspapers in the world were full of articles marketing the birth centenary of Srinivasa Ramanujan, Robert Kanigel asked: *Ramanujan Who ?* of an agent of the publisher (Charles Scribner's Sons) who was interested in bringing out a biography of Ramanujan. Inspired by the earlier writings on Ramanujan mentioned above, and intrigued by Hindu culture, Kanigel travelled to places that figured in Ramanujan's life in South India for five weeks.

[7]Based on the author's review of the book which appeared in *The Hindu*, Oct. 1993.

Kanigel interviewed the aging Janakiammal – the then living link to the legend – the relatives and friends of Ramanujan, and mathematicians. He was encouraged by the *Gang of three* mathematicians – George E. Andrews, the discoverer and propagator of the Lost Note Book; Bruce C. Berndt who took upon himself the arduous and challenging task of editing the three Note Books of Ramanujan containing about 3000 theorems without proofs since May 1977, and Richard Askey who spearheaded the commissioning of the bust of Ramanujan by Paul Granlund to present it to Mrs. Ramanujan. Kanigel brought out eventually (1991) a delightfully readable biography entitled: *The Man Who Knew Infinity: a life of the Genius Ramanujan* [X]. In this 438 pages book, Kanigel has intertwined into a braid three stories: the story of Ramanujan, the inscrutable intellect with a simple heart; the story of G.H. Hardy, the mathematician par excellence who not only recognized the genius *in the tattered garb in which it was clothed* but also strove to bring Ramanujan to the attention of the world and a silhoute of the mathematics of Ramanujan.

The book consists of eight chapters, a prologue, an epilogue and some forty pages of notes (in small print). The first three chapters deal with Ramanujan's childhood in the South Indian scenario, his early mathematical exploits and his search for patrons. These portions rely heavily upon the source material referred to earlier on. Kanigel has exercised the liberty of an experienced author to describe vividly the events of the period 1887-1913, as if he were an eye-witness.

He gives a good description of Kumbakonam, *the Cambridge of South India*, the home town of Ramanujan, flanked by the river Cauvery and one of its tributaries. Very little is said about Ramanujan's father. His mother is portrayed as a shrewd, cultured lady, whose obsession about Ramanujan's genius made her use her powerful personality to zealously protect and project his interests all through his life.

Kanigel agrees with E.H. Neville who referred to Ramanujan's early years, the five solid years after his academic failure, as the carefree days, before his marriage in 1909, and that in some ways, they were the most productive of his life. This is debatable in view of Hardy's observation that during

those formative years Ramanujan's *genius was misdirected, sidetracked and to a certain extent distorted.* Added to this is the fact that the excellent work published by Ramanujan from Cambridge, without recourse to his results written down in his Notebooks, including his collaboration with Hardy, could very well be considered as Ramanujan's most productive five year period in his life!

A brief description is given of the marriage of twenty-one year old Ramanujan to nine-year old Janaki, arranged by Komalathammal, not attended by Ramanujan's father, at Rajendram, in July 1909. Kanigel portrays the inauspicious incidents which marred the wedding, foreboding the tragic end to follow eleven years later, on April 26, 1920. Ramanujan and Janaki lived together only for a year in Madras before Ramanujan left for Cambridge in 1914, and for a year after he returned to India in 1919. *Scarcely off the boat, Ramanujan had dropped into the family snake pit*, states Kanigel. Komalathammal does not come out with flying colours through this last phase of Ramanujan's life when, like a tyrannical mother-in-law, she was quarreling with her daughter-in-law, tried her best to keep the couple separated, forcing Ramanujan to even rebel against her.

> *The family drifted back into something like the obscurity that was, save for the accident of Ramanujan's birth, its natural lot.*

Like in two of the four major novels of Sir Arthur Conan Doyle on Sherlock Holmes – viz. the *Valley of Fear* and the *Sign of Four* – Kanigel is in his elements when he starts his Chapter IV with a completely different venue and describes an Ivory Tower of British Science – Cambridge. The pivotal character whose stature was that of a mathematical colossus in the early decades of this century is none other than Mr.G.H. Hardy.

Kanigel provides a fascinating account of the venerable Cambridge institution called Tripos, which originally referred to a three-legged stool on which sat a man whose job was to dispute, sometimes humourously and sometimes aggressively, for the degree in mathematics. Tripos examinations for proficiency in mathematics consisted of two Parts. The disputant who *wrangled* over points of logic and who came at the top of the test of successful candidates was named a Senior Wrangler, an honour which clung to the recipient like an aura for a life-time.

Kanigel has vividly detailed the state of English mathematics and the turn given to it by Hardy[8], who chartered a course to the future and became a spokesman for the new rigor, with his text book *A Course of Pure Mathematics,* completed in 1908 – the first rigorous exposition of difficult mathematical concepts, in England.

It was at this stage of Hardy's life that the letter from the unknown Indian clerk reached him. Kanigel delightfully states that it was Hardy's willingness to stray from safe, familiar paths, his penchant for the unorthodox and unexpected, which enabled him to open his heart and mind to the wildly new and alien, mathematics-dense letter from India :

> *The wheels of Ramanujan's career, for ten years barely creaking along, now greased by Hardy's approval, began to whirr and whine like a finely tuned race car engine.*

The story is well told of how Hardy and Neville, the Cambridge mathematicians; Sir Francis Spring and Narayana Iyer, of the Madras Port Trust; Ramanujan's mother and her devotion to the goddess Namagiri of Namakkal; Lord Pentland, the Governor of Madras and his private Secretary, G.B. Cottrell; Professor Littlehailes and Francis Dewsbury, the Registrar of Madras University, in an orchestrated effort found the funds to send Ramanujan sailing to England, on a unique historic mathematical adventure of this century.

Kanigel's effort to make the mathematics of Ramanujan and Hardy accessible to the non-expert is highly commendable. The first two formulae in Carr's Synopsis and their generalization is illustrated; a generalization of the roots of the quadratic equation is given as an example of Ramanujan's weird notation, the convergence properties of infinite series, rational and irrational numbers, the ubiquitous nature of π stitched across the whole tapestry of mathematics, the definition of a modular equation, the distribution of prime numbers, the meaning of a complex number, Ramanujan's definition of a highly composite number, partitions of a number, the Rogers-Ramanujan identities, as well as the magic of some of Ramanujan's formulae sent to Hardy with his first letter, have all been generously interspersed with the stories of the lives of the two heros of this book – Ramanujan and Hardy.

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[8]This has been described in Chapter 4 : *Hardy on Ramanujan*

In Kanigel's rendering Ramanujan emerges as one untouched by the seductive charms of the West, successfully labouring under an impossible handicap – *a poor Hindu pitting his brains against the accumulated wisdom of Europe* – self-driven, self-willed, every bit the product of India and its customs; holding strong religious beliefs; impetuous when overcome by shame; finicky and demanding in sickness; warming to the kindness of Hardy; who was not discovered by Hardy or anyone else but *like other great men he invented himself:* a Swayambhu or *self-born* genius.

Kanigel states that the 140 pages of material pertaining to *'mock'* theta functions, discovered in 1976, and named as the *'Lost' Notebook* of Ramanujan by George Andrews, was part of a batch of papers Dewsbury had sent Hardy in 1923 and that in 1928 Hardy handed over the notebook and manuscripts of Ramanujan to G.N. Watson – an explanation for how the 'Notebook' was 'lost' but found by Andrews in the estate of Watson.

In the Epilogue Kanigel completes the story of Hardy, who won several honors and who died on December 1, 1947, the day he was to be presented the Copley Medal, the Royal Society's highest honour.

The forty pages of Notes giving the sources for the quotations and statements contain interesting snippets and show the thorough analysis of all the material digested by Kanigel. This separation of the Notes, without markings in the text, contributes to the smooth flow in the reading of this delightfully written first full-length biography of Ramanujan.

Scribner's first publication was priced at $ 27.95. Soon, Rupa & Co brought out an Indian edition which costs much less (Rs. 195/- for the paper back edition).

Review of Berndt and Rankin's book (1995)

Ramanujan: Letters and Commentary by Bruce C. Berndt and Robert A. Rankin [XI] is a classic work containing the first letter from Ramanujan to Hardy, "*one of the most famous letters in the history of mathematics*", and Ramanujan's last letter – containing until the discovery of the 'lost' Note-

book, "*the only information available about Ramanujan's work in 1919 – 1920*". The authors have left no stone unturned to acquire as many letters as possible, since most of these letters, especially those exchanged by Hardy and Ramanujan (when the latter was for about two years in different sanatoria in England) contain interesting mathematical results. In fact, the two Memorial Numbers *Ramanujan: Letters and Reminiscences* [I] and *Ramanujan: an Inspiration* edited by P.K. Srinivasan and published by Muthialpet High School, Madras, in 1968, are the precursors to this book by Berndt and Rankin.

After a brief Biography of Ramanujan, transcripts of 172 letters are reproduced with Commentaries at the end of each of them, by Berndt and Rankin, in eight chapters, in this book whose Special Indian edition (1997) has 363 + xviii pages and is prices at Rs. 295/-. In Chapter 1, the letters regarding Ramanujan's first position at the Madras Port Trust and the correspondence with Prof. M.J.M. Hill (referred to in Chapter 1 of this book) are reproduced. In Chapter 2, the first two historic letters of Ramanujan to Hardy are reproduced in full and the commentary is on each one of the formulae contained in them. This is the first time that the second letter of Ramanujan to Hardy appears in full. In the Collected Papers [III] there is missing material marked by ... at certain places. There are brief biographical sketches (of a few lines or a few paragraphs) of almost all the characters who wrote to Ramanujan or who are mentioned in his letters in this book. Hardy's list of six New Year wishes, contained in a post card to a friend of his in the 1920's is an example of a delightful snippet in one of the Commentaries [XI, p.33] :

> (1) *prove the Riemann hypothesis*; (2) *make 211 not out in the fourth innings of the last test match at the Oval*; (3) *find an argument for the nonexistence of God which shall convince the general public*; (4) *be the first man at the top of Mt. Everest*; (5) *be proclaimed the first president of the U.S.S.R., Great Britain, and Germany*; (6) *murder Mussolini.*

The next four chapters are devoted to the letters pertaining to Ramanujan's *Preparing to go to England*, *Ramanujan at Cambridge*, Ramanujan's illness and those written after Ramanujan returned to India. Berndt and Rankin have brought out in this volume, for the first time, Ramanujan's letters from sanatoria (Matlock House, Hill Grove, Fitzroy House). Ramanujan who was

confined to sanatoria off and on for nearly half the time of his five-year sojourn in England, wrote several notes/letters to Hardy and received notes/replies from Hardy. These contain mathematical insights into the problems of interest to the two of them. These letters and commentaries on them, therefore, make this book significant to the mathematicians. The letters of Ramanujan to his kith and kin and friends reflect the human angle of Ramanujan. The incisive commentaries by the editors provides a proper perspective of the persons with whom Ramanujan corresponded. The last letter written by Ramanujan to Hardy, on Jan. 12, 1920 is reproduced and a portion of the letter following the first paragraph of the letter has been missing [XI, p.223-224]:

> *For almost 60 years, the only information available about Ramanujan's work in 1919 – 1920 was contained in his last letter to Hardy. Now we know that the results Ramanujan described in this letter are part of a much longer collection, called the "lost notebook"* [XI].

In this letter, Ramanujan (as pointed out earlier) did not even hint to Hardy that he had not only not recovered from his suspected tuberculosis but that his health was on the decline. However, it contained the heart-warming news to Hardy that he had found new results on 'mock' theta functions.

The minor role I had in providing the editors with letters written by Ramanujan to his boyhood friend, Mr. E. Vinayaka Row, is explicitly stated in Chapter 4 of the book [XI, p.113] :

> *The three letters below that Ramanujan wrote to Mr. E. Vinayaka Row on 11 June 1914, 24 march 1915 and 10 September 1915 have been made available through the courtesy of Professor K. Srinivasa Rao of the Institute of Mathematical Sciences, Madras. They were given to him by Mr. V. Madhava Rao, the son of E. Vinayaka Row. The latter was a lawyer and resident in Mylapore and a friend of K. Srinivasa Rao's father. The letters, parts of which have become somewhat moth-eaten, were discovered in a suitcase in Mr. Madhava Rao's attic. ... Hardy first reported Ramanujan's illness in the spring of 1917. In this letter [dated 24 March 1915], we learn that already during Ramanujan's first winter in England he was not feeling well.*

The originals of these letters have since been handed-over by Mrs. Saraswathi

Dole (daughter of Mr. E. Vinayaka Row) to the Ramanujan Museum in Royapuram, Madras[9].

In Chapter 7, the letters announcing the death of Ramanujan to Hardy, settling of the balance amount in the account of Ramanujan with the Barclays Bank Ltd. in Cambridge, correspondence between the relatives of Ramanujan (his mother and his wife) wrangling for the pension with the Government of Madras and its resolution, and letters by Hardy to the University of Madras and the Port Trust are reproduced. In Chapter 8, the letters pertaining to the publication of the Collected Papers and manuscripts of Ramanujan, and some correspondence between Hardy and G.N. Watson and from G.N. Watson to B.M. Wilson are dealt with. In Chapter 9, a very brief account of the Family history and the Family Record of Ramanujan with the National Archives in Delhi are presented.

Berndt and Rankin were interested in getting an Indian Edition of this book originally published by the American Mathematical Society and the London Mathematical Society, in 1995, and they graciously permitted me to write a Preface to the Indian edition and include a few Additions to the Indian Edition, published by the Affiliated East West Press Ltd. (1997) [XI]:

> *In this Indian edition, which coincides with the second printing of this book published by the American Mathematical Society (AMS) and London Mathematical Society (LMS), an Errata to the first edition is appended at the end of this volume, in a section entitled Additions to the Indian Edition. The letter of Mr. E.S. Montagu, referred to as No.22 – Financial dated February 24, 1921, in the AMS-LMS edition, and a xerox copy of Ramanujan's letter to Mr. E. Vinayaka Row (one of three reproduced, on p. 113 - 117) now deposited with the Ramanujan Museum in Royapuram, are included. ...*
>
> *To the excellent collection of photographs in the AMS-LMS edition, are added three photographs of the busts of Ramanujan: that made by Paul Granlund, that by N. Masilamani and that on the beautiful white marble Saraswathi temple tower ('gopuram') of the Birla Institute of Technology (BITS).*

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[9] A brief description of the Museum is given in Chapter 7 of this work.

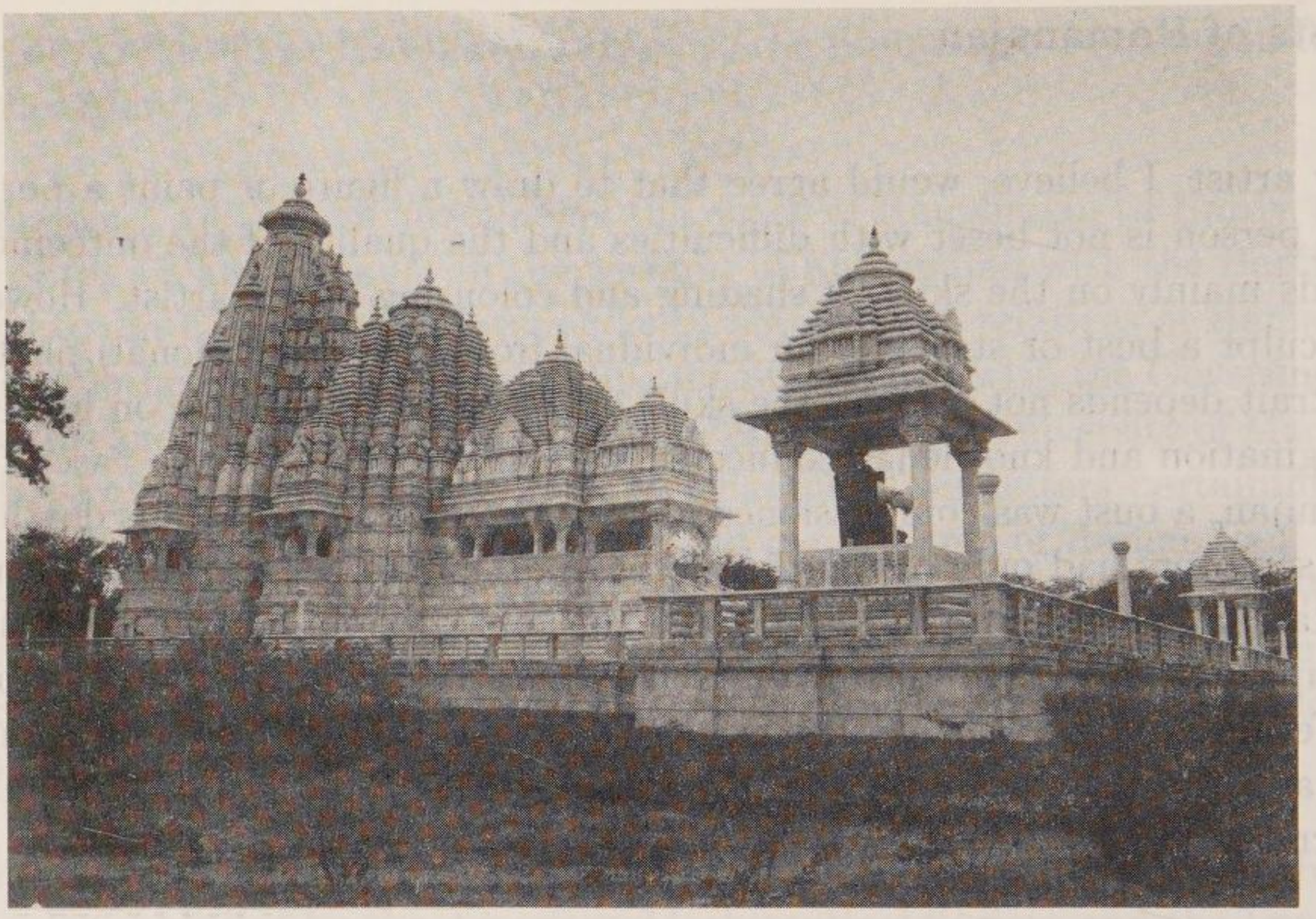
Busts of Ramanujan

Any artist, I believe, would agree that to draw a figure or paint a portrait of a person is not beset with difficulties and the quality of the outcome depends mainly on the skills of shading and colouring of the artist. However, to sculpt a bust or statue of an individual from a (2-dimensional) photo or portrait depends not only on the skills of the sculptor but also on his fertile imagination and knowledge connected with the subject. In the case of Ramanujan, a bust was commissioned almost seven decades after his death and the sculptors had only one authentic passport photo (in addition to photos of Ramanujan either in the ceremonial garb of a graduate or with others in a group [I, II, XI]). The bust on the Saraswathi Temple in the Birla Institute of Technology and Science, Pilani, perhaps the earliest sculpted of Ramanujan and the later busts sculpted by Paul Granlund and N. Masilamani are described in this section.

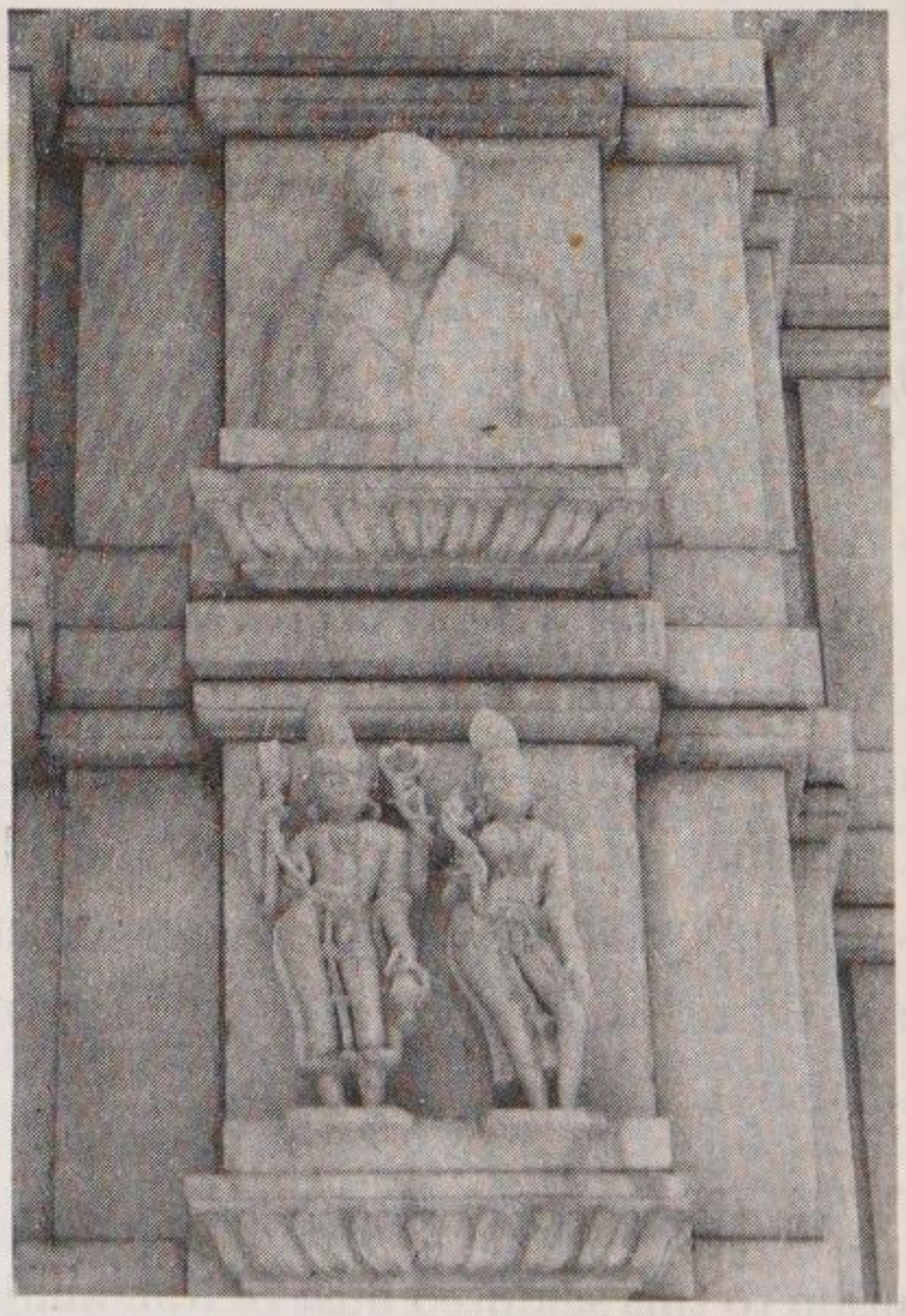
On the temple tower of BITS, Pilani

While there are many institutions in India where different portraits of Srinivasa Ramanujan can be found framed, perhaps the first sculpted bust of Ramanujan is the one found on the Gopuram ('temple tower') of the Saraswathi (goddess of learning) Temple in the Birla Institute of Technology and Science (BITS), Pilani, Rajasthan. The work on this beautiful white marble temple dedicated to goddess Saraswathi, called the Sharada Peeth, was started in January 1956 and was completed in June 1959. The temple was consecrated by Mr. Morarji Desai on February 2, 1960. The imposing marble Gopuram is 109 feet in height and there are in all 959 idols and statues on the outer Prakara ('wall'), at the base of the temple tower, made by Shri Narain Mal Jaimini of Jaipur[10]. The bust of Ramanujan is one of several in the middle row of the three rows along with those of other eminent persons like: Sir C.V. Raman, Mahatma Gandhi, Rabindranath Tagore, Jawaharlal Nehru, John F. Kennedy, Vladimir I. Lenin.

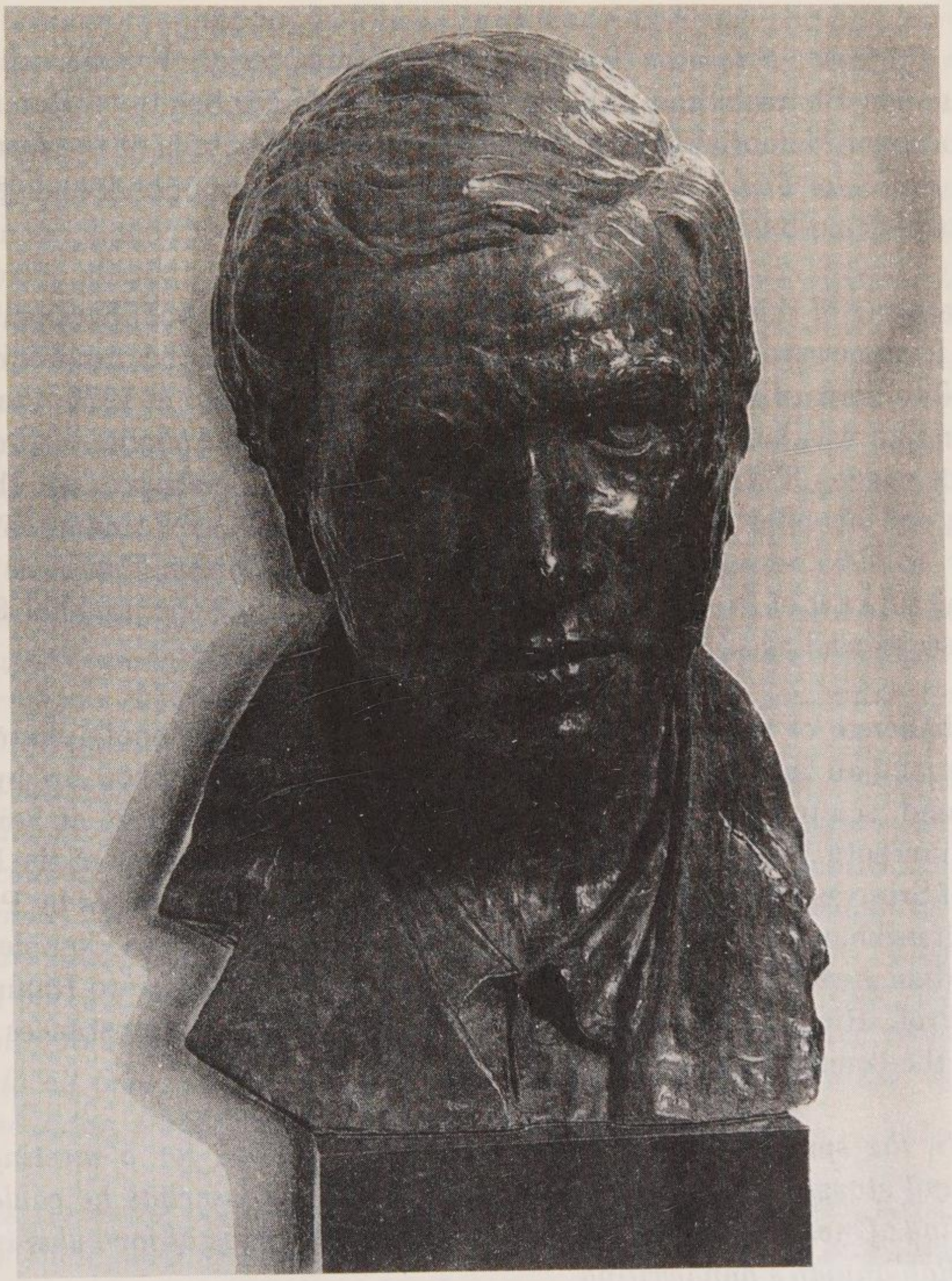
[10]Thanks are due to Dr. S. Venkateswaran, Professor of Mathematics and the Director of BITS for drawing our attention to this sculpted bust of Ramanujan and to Dr. K.R.V. Subramanian, Dean, Distance Learning Programme Division, BITS, for details about the Saraswathi temple.



The Saraswathi Temple in B.I.T.S., Pilani.



The sculpted figure of Ramanujan on the temple tower.



Ramanujan by Paul Granlund.

Bust made by Paul Granlund

In the *Professor Srinivasa Ramanujan Commemoration Volume*, edited by Ramananda Bharathi and published by the Professor Srinivasa Ramanujan International Memorial Committee, Madras (1974), details of the formation of the *Professor Srinivasa Ramanujan bust Committee* can be found (pp. ix–x). However no bust was made.

In Chapter 1 of this book, it has been pointed out that Prof. Richard Askey learnt from newspaper reports which appeared soon after the discovery of the *Lost* Note book of Ramanujan by Prof. George Andrews, in 1976, that Mrs. Ramanujan was lamenting the absence of a bust of Ramanujan – promised but not made. This was sufficient for making Prof. Askey to set the ball rolling and his efforts culminated in presenting a bust of Ramanujan to his widow, in 1985, at a function in the University of Madras. Professor Askey had the support of Professors George Andrews and S. Chandrasekhar for this purpose, as well as several mathematicians and scientists.

The challenge of transforming the passport (two-dimensional) photograph of Ramanujan into a three dimensional bust has been taken up by Paul Granlund, sculptor-in-residence at Gustavus Adolphus College at Saint Peter, Minnesota, U.S.A. On Feb 6, 1985, while presenting a copy of the bronze bust of Srinivasa Ramanujam to the Indian Academy of Sciences by Prof. S. Chandrasekhar and Mrs. Lalitha Chandrasekhar (ref. Patrika, Newsletter of the Indian Academy Sciences, No. 10, April 1983), a tribute to Ramanujan from Prof. Richard Askey was read out, detailing the circumstances which led to the commissioning of the bust:

> *In the spring of 1976, Andrews went to Europe for a meeting and stopped in Cambridge to see what old manuscripts he could find. One find was not a manuscript but 140 pages of formulas in Ramanujan's handwriting.*
> *The story of the thread from these sheets to the bust is simple. Andrews has done a lot of very deep work trying to understand what Ramanujan discovered. Eventually 'The New York Times' heard about it and interviewed him. 'The Hindu' followed with a more extensive interview and also published an interview with*

Ramanujan's widow, Janaki Ammal. She lamented the fact that a statue of Ramanujan had never been made, although one had been promised. Andrews sent me copies of these interviews, and after a couple of months my subconscious mind finally got through to my conscious mind and it was clear that a bust should be made. Since Janaki Ammal was 80, time was important, so it was up to individuals rather than governments or societies, since institutions move slowly. My first reason for wanting a bust was simple; if Ramanujan's widow wanted one she should have it. That was the least we could do to show our appreciation of Ramanujan, and while she did not understand his mathematics, she knew that he was one of the few whose work will last. As long as people do mathematics, some of Ramanujan's work will be appreciated. Fame is a strange thing and is often fleeting. An interview on a television program is now the accepted form of honour.

In Ramanujan's case a permanent memorial is appropriate: one which can be appreciated by those who do not understand his mathematics should be added to the memorial Ramanujan made for himself with his work.

I am pleased to have played a role in this, and would like to thank the more than one hundred mathematicians and scientists who contributed money for the bust which was presented to Janaki Ammal. The bust being dedicated today was donated by a couple who are now friends, Subramanyan and Lalitha Chandrasekhar. When I asked Chandra about the appropriateness of a bust of Ramanujan, he immediately replied that it was a good idea and they would do all they could to help. They did. Finally I want to thank the sculptor, Paul Granlund.

Besides the proof copy for the sculptor, ten copies of the bust were made. Two of these castings were acquired by the Chandrasekhars and these were gifted by them to the Indian Academy of Sciences and to the Cambridge University. At present[11], five of the busts of Ramanujan are in India, two in England and four in the United States of America. The five copies in India

[11] I am thankful to Prof. Richard Askey for the information about the whereabouts of the copies of the bust of Ramanujan made by Paul Granlund.

are with:

- the foster son of Mrs. Ramanujan – the copy presented to Mrs. Janakiammal in 1985 – at 14, Hanumantharayan Koil Street, Triplicane, Madras.
- the Indian Academy of Sciences, Raman Research Institute, Bangalore.
- the Ministry of Defence, Government of India, New Delhi.
- the Tata Institute of Fundamental Research, Mumbai.
- the Inter-University Center for Astronomy and Astrophysics, Pune.

The two copies in England are with:

- the Pure Mathematics Library, Cambridge University, Cambridge.
- the Royal Society, London.

Of the four copies in the United States of America, two are in the private collections of individuals and two are with:

- the Gustavous Adolphus College, St. Peter, Minnesota.
- the Vaughan Foundation, Houston, Texas.

Bust made by N. Masilamani

On the occasion of the birth centenary of Srinivasa Ramanujan, on Dec. 22, 1987, Dr. B. Ramamurthy was requested[12] that a fitting memorial should be set up for the 'natural genius' in Madras and an appeal should be made for artefacts and letters pertaining to Ramanujan, so that these could be preserved along with his original Notebooks in a Museum. As a consequence, Dr. B. Ramamurthy, the eminent Neurosurgeon of India, who was then the acting Vice-Chancellor of the University of Madras, set up a Committee, had a Syndicate resolution passed and made a public appeal in the newspapers. The Ramanujan Museum

[12]By the present author.

Committee formed at that time, since got a bust of Ramanujan commissioned and made by Mr. N. Masilamani, the son of Mr. S. Nagappa. Mr. Nagappa was a renowned artist, painter and sculptor of Madras, whose works of art (more than 100) can be found[13] at the Theosophical Society's head quarters in Madras, in the High Court of Madras, at the Ripon Buildings and in several cities in India. Mr. Masilamani belongs to this school of Nagappa, a hereditary Sthapathigal ('sculptor') family and has been trained by his father, who was felicitated by the Government and other agencies with several gold medals for excellence in arts.

[13]One of these, a majestic bronze lion was acquired by my father, (late) Mr. K. Vallabheswar Rao, who was a legal advisor to Mr. S. Nagappa.

This bust of Ramanujan was unveiled on March 26, 1993, on the occasion of the inauguration of a Special Symposium on 'Approximation Theory and its Applications', convened by Professor Geetha Srinivasa Rao and organized as part of the Silver Jubilee (of the University Grants Commission conferring an Institute for Advanced Study status to the Department of Mathematics of the University of Madras and merging it with the Ramanujan Institute founded, in 1950, by Mr. Alagappa Chettiar) celebrations of the Ramanujan Institute for Advanced Study in Mathematics. It is now permanently housed in the foyer of this Institute, in Chepauk, Chennai.

7. What is where

The Notebooks of Ramanujan and the manuscripts concerning him are now scattered in different Libraries and Archives. The publications of his *Collected papers* [III], Hardy's lectures on *Ramanujan* [IV], the facsimile editions of his Notebooks [VII] and Bruce Berndt's five Part publication on *Ramanujan Notebooks* [VIII] are undoubtedly the most valuable collection of the mathematical works of Ramanujan and most reputed research institutions the world over have them in their libraries. In this section a brief account is given of where the original documents are.

Ramanujan Notebooks

Let us recall the history of the Notebooks: Ramanujan had noted down the results of his researches, without proofs, (as in the *Synopsis of Elementary mathematical Results*, by G.S.Carr), in three Notebooks, between the years 1903 - 1914, before he left for England. These were the Notebooks which he showed to his benefactors to convince them about his abilities as a Mathematician. The results in these Notebooks were organized by him. The first Notebook has 16 Chapters in 134 pages. The second is a revised, enlarged version of the first, containing 21 chapters in 252 pages. The third Notebook contains 33 pages of unorganized material. Ramanujan took these Notebooks with him to Cambridge. But, in one of his letters to a friend, he wrote that he had no time to look into them and most probably he did not put them to use during his five year stay abroad.

When Ramanujan left for India, he left behind one of his Notebooks with Prof. Hardy. Three days after Ramanujan died, his brother S. Lakshminarasimhan, in a letter to Prof. Hardy, dated 29th April 1920, informing him about his demise, wrote:

> *All his Mss that were in his trunk were handed over on the day of his death to Mr. Ramachandra Row's son in law, since the former is at Nilgiris. Not only those but also the journals, magazines - all he possessed except the books which he had, were taken away by them.*

In a letter to Prof. Hardy, dated 3 Dec. 1920, Mr.R. Ramachandra Row, wrote:

> *Ramanujan's M.S.S whatever they are are with me and will be handed over intact to the University of Madras who I understand is already in correspondence with you regarding the methods of publication.*

After Ramanujan's death, Hardy spent three to four months on a Chapter in the Notebooks, on Hypergeometric series, and in the introduction to the paper he published on this topic he wrote: *a systematic verification of the results would be a very heavy undertaking.* He tried to persuade the University of Madras and others to undertake such a task. He passed on the Notebooks to Prof. G.N. Watson and Prof.B.M. Wilson, who divided the task of editing these Notebooks and started working on their parts. However, the premature death of Prof.B.M. Wilson put an end to this effort.

In March 1925, Professor Hardy gave the first Notebook of Ramanujan with him to Dr.S.R. Ranganathan, the then Librarian of Madras University, with the remark ([VI], p.57): *Ramanujan belongs to your country, the proper place for his Notebook is your own (Madras) University Library.*

The second and third Notebooks of Ramanujan were donated to the Madras University Library after his death. T.A. Satagopan at the University of Madras made three hand written copies of all three Notebooks and one copy of each of these was sent to Hardy.

In 1957, at the instance of Prof. K. Chandrasekhar and Prof. Homi J. Bhabha, the Tata Institute of Fundamental Research, Bombay, brought out the facsimile edition of these Notebooks in two volumes, without any commentary. These were reprinted [VII] by Narosa Publishing House in 1987.

A resurgence in the work of Ramanujan occurred, in the Spring of 1976, when Prof. George E. Andrews, of the Pennsylvania State University, while going through the Estate of Prof. G.N. Watson (who died in 1965) with the Trinity College, made the historical discovery of 140 pages of Ramanujan's papers in a box, containing some 600 theorems mostly on mock theta functions. These were results which Ramanujan had noted down during the year when he was deathly ill, in India, after his return from Cambridge. To these papers, Prof. Andrews gave the romantic name, the 'Lost' Notebook of Ramanujan and wrote a series of articles [53, 56] in the American Mathematical

Society. He set for himself the task of proving several of the theorems. The 'Lost' Notebook was published and released along with the reprints of the Notebooks of Ramanujan, by Narosa Publishing House on the occasion of the Ramanujan birth centenary celebrations, held in Madras, in Dec. 1987.

Of course, there have been many individual Mathematicians, all over the world, who worked on some aspects of the results in the Notebooks and several hundreds of research articles were published. A computer search by Bruce C. Berndt revealed that between 1973 – 84 over 200 research papers refer to Ramanujan in their titles or abstracts. But there was no concerted or dedicated effort, in the direction as suggested by Hardy, till the 1980s. Prof. Bruce C. Berndt, at the University of Illinois, Urbana-Champagne, took up the *heavy undertaking* or *formidable task* of editing the Notebooks of Ramanujan in right earnestness and set for himself the goal *to prove each of Ramanujan's theorems*, if they were new and for the known results refer to literature where proofs may be found. The details of the contents of the Notebooks and other Manuscripts is now recorded in the authentic work of Bruce C. Berndt [VIII].

The original Notebooks are now in the main Library of the Madras University (in Chepauk) and are not easily accessible. The pages are stuck in between tanslucent paper and this should have been done at a time when lamination of the pages was not possible. A few tracks indicating moth holes are beginning to appear. The Notebooks should not be left in this condition and should be better preserved. They are invaluable possessions of this country.

The Wren Library

King's College, Trinity College and St.John's College of Cambridge University are next to each other on Kings parade, which narrows into Trinity Street. The spacious quadrangles with lush green and velvety lawns behind the picturesque main facades of these Colleges come as a total surprise to any visitor, who would also be greeted by numerous 'PRIVATE', 'NO ENTRY' sign boards, warding him off from the premises. These magnificient court yards are open to public for two hours on working days (12 Noon to 2 p.m.). The Visitors are requested to be conscious of the studying atmosphere and the fact that some of the quadrangles are also residences of the scholars.

These three Colleges and the Queen's, Clare and the St.Catherine's Colleges have the willow-tree lined curving Cam river flowing through great green pastures at their rear. A suggested walk in Cambridge (for about one and a half hours) takes one through these tradition conscious institutions, as well as the majestic King's College Chappel, criss-crossing six bridges and one can also see a seventh one called 'Mathematical Bridge', an intricate wooden bridge, spanning the river Cam at Queen's College.

"Henry VIII took over two colleges [the King's College and Michaelhouse, of the 14th Century] *to create Trinity College in 1546, giving it substantial endowments from monasteries he had dissolved. But its present grandeur is largely due to Thomas Neville, Master from 1593, who created the Great Court. The cloistered Nevile's Court was later completed by the building of Sir Christopher Wren's magnificient Library. The largest of Cambridge Colleges, Trinity College, boasts some twenty-eight Noble Prize-winners*, proclaims a guide book [73] on the University City of Cambridge.

The Master of Trinity College was the reputed Mathematician Sir Michael Atiyah, F.R.S., in 1995. Like most of the 31 Colleges in Cambridge, Trinity has (about 660) undergraduate students, (300) graduate students and some (140) Fellows. Some of the renowned Trinity men are : Byron, Dryden, Macauley, Tennyson; Sir Isaac Newton, James Clerk Maxwell, Lord Rutherford, Bertrand Russell, Jawaharlal Nehru, Sir Arthur Eddington, S. Chandrasekhar, G.H. Hardy, J.E. Littlewood and Srinivasa Ramanujan. While the Cambridge University is a central body which provides coordinating administration, lectures, laboratories, libraries, examinations and degrees, the 700 year-old Colleges provide educational supervision, tuition and residence for its members. However, the Colleges are separate foundations having their own libraries and are largely autonomous.

The Wren library [74], completed in 1695, is the older part of the Trinity College library, named after its designer Sir Christopher Wren, containing the collection as in 1820 with a few later additions. Its internal size is 191 ft. by 40 ft. and has a 38 ft. high ceiling. Four statues on the roof of the Wren Library represent Divinity, Law, Physics and Mathematics. The interior has carvings in lime-wood and oak panellings. The stained glass window at the Southend of the Wren Library represents Sir Isaac Newton being presented

by lady 'Fame' to King George III, with Francis Bacon in the foreground. At this end are also the marble busts of Bacon and Newton. Twenty-eight busts on the top of the wooden book cases are plaster casts of ancient and modern writers (most with no connection with Trinity). A full-length statue of Byron on a high pedastel is near the southern end and beyond this are the (eight) Reader's places[1].

The Wren Library has a total stock of about 300,000 volumes of which 2500 are early manuscripts - 1250 medieval ones, 750 are 15th century printed books - 75,000 are pre-1820 printed books. It also has the Capell collection of Shakespeariana, many books from the library of Sir Isaac Newton and the Rothschild collection of 18th century English literature.

Visitors to the Library breath in the air of the Wren Library and view some of the rare exhibits placed in glass covered tables: the first bound volume of the Plays of Shakespeare, the handwritten sonnets of Milton, some beautifully illustrated pages of Pilgrim's Progress, an open page from Winnie the Pooh, as well as the 8th century manuscripts of the Epistles of St. Paul, are some of the displayed exhibits.

With the help of Prof. R.A. Rankin, the Wren Library has preserved and catalogued the papers of Ramanujan. These have the reference addresses as $\underline{Add.Ms.a.94}$, with a superscript which refers to the specific document, and as $\underline{Add.Ms.a.94^{1-18}}$, are contained in large sized (A3-size) 2 in. high cardboard box. The references $\underline{Add.MS.b.100-107C}$ refer to the five bound volumes (fullscape size) and, in addition to these, the papers and letters are preserved in two large (A3) sized thick card board boxes. In Appendix 2 are given the details of the Ramanujan MSS as maintained in the Archives of the Wren Library of Trinity College. In fact, Appendix 2 is a more detailed version of the Supplement II in Venkatachaliengar's article [75].

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[1]Professor R.A. Rankin of Glasgow University – whose 80th Birthday was celebrated with a two day conference on the 27th and 28th of October 1995 – when requested, readily wrote to Dr. David McKitterick, the Librarian of Trinity College and obtained for me a Reader's Place for two days (Oct. 17 and 18, 1995) in the hallowed Wren Library. This introduction provided me easy access to the invaluable treasures preserved by them regarding Ramanujan.

In 1980, Prof. K. Venkatachaliengar [75] quoted Hardy's remarks to Prof. S.R. Ranganathan – viz. *Ramanujan belongs to your country, the proper place for his Notebook is your own University Library* – and added:

> *Naturally what applies to Ramanujan's Notebook applies equally to the other manuscript material which were passed on by Prof. Hardy in subsequent years to Professor Watson. I request our younger colleagues and the Madras University to request the Universities of Oxford and Cambridge to deliver all the Ramanujan manuscripts to our country keeping photostat copies of all of them in England. This would naturally take time; hence I request the University Colleges and institutes of higher learning in our country to obtain photostat copies of the manuscripts at Oxford and Cambridge [The cost may be a couple of thousand rupees.] and make them available to interested scholars of our country.*

In his Presidential address to the Forty-fifth and Forty-sixth Conferences of the Indian Mathematical Society, held at Ranchi (Dec. 1979) and at Bangalore (Dec. 1980), respectively, Prof. K. Venkatachaliengar [75] spoke on the *Ramanujan Manuscripts*. His address in 1979 ended with the plea:

> *The task of getting the originals or at least photo copies of all the Ramanujan manuscripts must be undertaken by our countrymen, who should also face the formidable task of editing them, taking at first those parts which are comparatively easy to edit; this is true of his elliptic function formulae which Ramanujan has given in nearly half of his note books. ... May I hope that the University Grants Commission will engage scholars to perform this task as early as possible.*

In Appendix 3, we give the papers of Ramanujan passed on by the Madras Port Trust to the National Archives at New Delhi where they are preserved.

A scrutiny of the Appendices 2 and 3 will inevitably reveal that the most interesting material is in the Wren Library but then through the book *Ramanujan: Letters and Commentary* [XI], Bruce C. Berndt and R.A. Rankin have made this public. The originals of the Ramanujan Notebooks are with

the Madras University and so is Carr's *Synopsis*. Before we think of approaching the Wren Library to part with their valuable treasures we must have a suitable place to house them, under controlled conditions of temperature and humidity. Till then, the Ramanujan Papers are better off where they are! Let us record the first efforts in this direction.

Ramanujan Museum

In 1962, on the occasion of Ramanujan's 75th birth anniversary, Mr. P.K. Srinivasan, an ardent, enthusiastic Mathematics Teacher then at the Muthialpet High School in Madras, started an effort, with courage, conviction, enthusiasm and earnestness, which culminated in the release, in 1968, of *"Ramanujan: Letters and Reminiscences"* and *"Ramanujan: an Inspiration"*, two Memorial Volumes [I,II] containing the first ever collection concerning Ramanujan. In an editorial to the first of these two volumes, Mr. P.K. Srinivasan wrote about the formation of a

> *Ramanujan Memorial Foundation, with an object of setting up a permanent memorial to Ramanujan in the shape of a multi-storeyed building in Madras, housing a planetarium, mathematics exhibition wings, auditorium, library and show rooms displaying applications of mathematics in Industry. It will be a house of entertainment 'par excellence' for the layman and it will strive to make mathematics as popular as dance and music.*

That dream of his is yet to come true and what he has been able to achieve to-date is a room set apart as the Ramanujan Museum in the Avvai Kalai Kazhagam, in Royapuram, Madras. The origins of the Museum – *a museum with a mission* – is best described in what Mr. Srinivasan, its Founder Resident Curator, said on April 26, 1996, the Remembrance Day of Ramanujan:

> *Way back in May 1968 when Rajaji released the 2 volume Ramanujan Memorial Numbers [I,II] at Gokahlae Hall, it was announced that there should be a museum to inspire people with the creative genius of Ramanujan and his singular life of dedication to mathematics. The announcement was received with thunderous applause.*

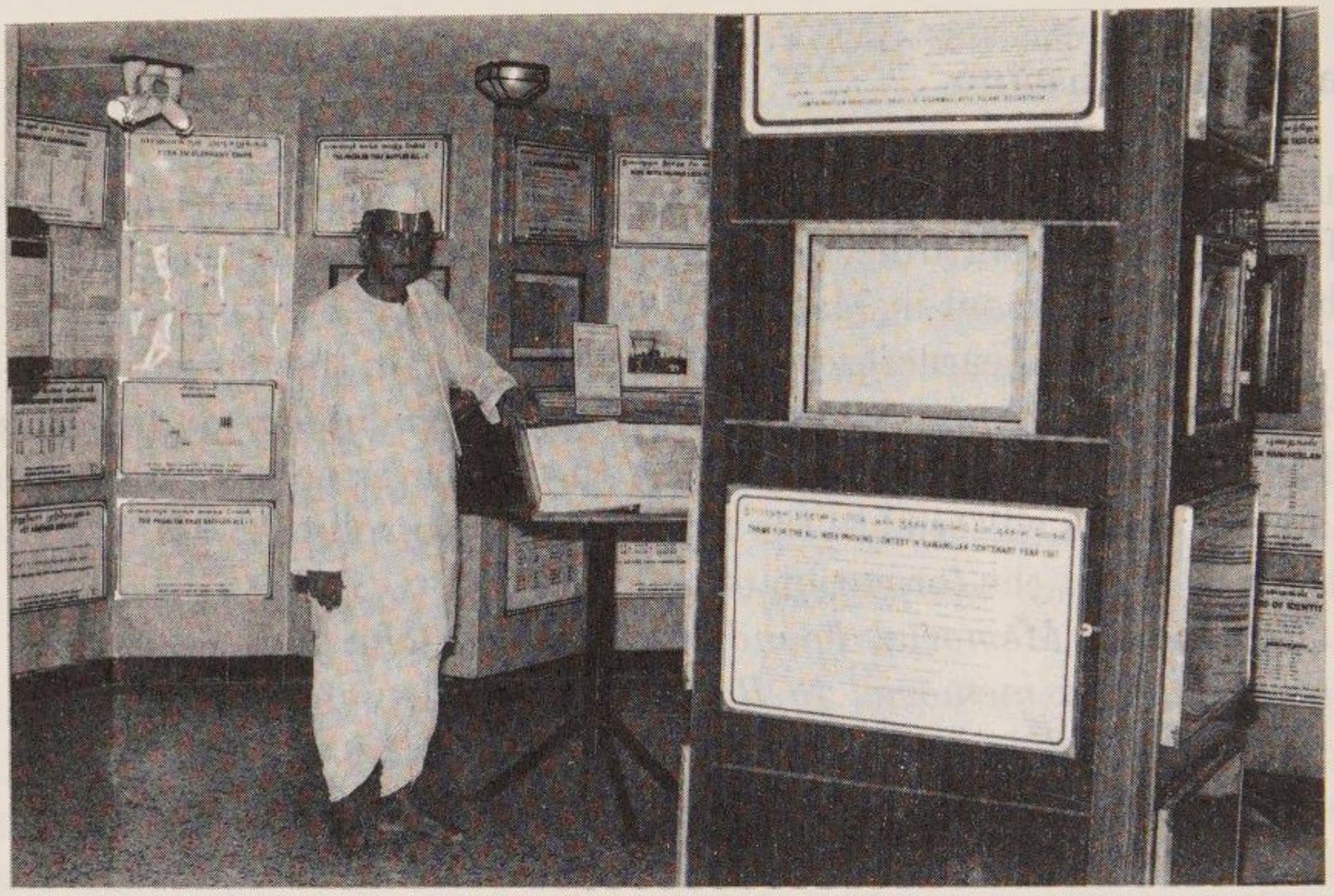

Fig. caption: The Ramanujan Museum, Royapuram, Madras with its indefatiguable Founder Curator: Mr. P.K. Srinivasan.

Since 1968, all the attempts to find a patron for housing the museum did not bear fruit. Nor was any museum started. Avvai Academy, a premier organization founded in 1992 in North Madras with its redoubtable secretary Mr. A.T.B. Bose came forward to give the museum a place as one of its units. On 18th March 1993, the Museum was declared open by our elder statesman Sri C. Subramanian, who has been awarded the country's highest honour of Bharath Rathna in Feb. 1998.

It would be appropriate to observe that the Museum is still in the making and it presents Ramanujan as a mathematician in the making of a super student star. The museum started with a few

original and xeroxed letters, more than 8 decades old, of and to Ramanujan in their facsimile edition, panels of photographs of buildings associated with Ramanujan, of foreigners who played a role in the recognition and promotion of research career of Ramanujan and of Indians who had something or other to do in his passage from obscurity to fame.

With handsome financial assistance of The Hindu, *thanks to Mr. N. Ravi, distinguished editor who readily granted the request of Srinivasa Ramanujan Museum Associates represented by Sri. S. Balathandapani of National Law School, a former student of the curator, sixty one custom made commercial art produced, laminated and framed bilingual (Tamil & English) charts of uniform size were procured and placed in the museum in 1995. They contain the jottings of interest to school students – jottings culled by the curator from his Notebooks. This became the second phase.*

Schools have started sending their children to the museum. XI standard Math students simply marvel at the creative contribution of Ramanujan even at School mathematics level and enquire why they were not included in textbooks and taught in the class room.

It is gratifying to make it known that Prof. C.G. Swaminathan donated in August 1994 one original letter of Ramanujan to his late father Prof. C.N. Ganapathy Iyer of Presidency College. This was an important addition to the museum as it tells us where Ramanujan saw Prof. Hardy's book: Orders of Infinity *that triggered their correspondence.*

Now we have gathered to witness on this 76th Remembrance Day of Ramanujan a historic event of Mrs. Saraswathi Vasanth Dole donating to the museum three original letters written by Ramanujan[2] *to her late father Mr. E. Vinayaka Row, thanks to*

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[2]The Museum has besides these three letters the following five **originals**: two of Hardy's letters, dated Feb. 8 and March 26, 1913, written in repsonse to the historic

the recommendation of Prof. K. Srinivasa Rao of MATSCIENCE.

... Announcement has already been made regarding the need to launch a massive 10 acre 10 crore 10 year project for setting up a Ramanujan Mathmagic Plaza *of international standard that will not only portray Ramanujan's genius with all its splendour but also present the vast expanse, the great beauty and the tremendous power of mathematics, Today and Tomorrow. This alone will do full justice to the great memory of Ramanujan whose stature is still rising to the wonderment of the entire mankind.*

... Attached to the museum an active Math Education Center is functioning to take up the neglected areas of math education such as teacher growth through credit courses in intensive practicals, indigenisation of math curriculum through incorporation of Kolam and folk mathematics, inter school primary math olympiad, help to schools in organizing math club, math lab and math expo, in building up the math section of the libraries and display of wall paintings of mathematical interest, publication of recreational math books and alternative instruction strategies for teachers. Culturisation of mathematics through greeting school days with date magic square, magic square dances and inclusion of mathematical themes in school annuals and school dramas, production of math learning kits, summer math camps, reference library for journalists, etc.

The stature and achievements of Srinivasa Ramanujan, hailed as *one of the greatest Mathematicians of the 20th Century,* should be an eternal source of inspiration for students of Science. This can best be achieved through the setting up of a **National Science Museum**. Like the Sistine Chapel of the Vatican, pointers in such a Museum should be to the original Notebooks of Ramanujan in its *sanctum sanctorum*!

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first two letters of Ramanujan dated Jan. 13 and Feb. 27, 1913; two letters written by Ramanujan to his mother (dated 11 Sept. 1914) and father (dated 17 Nov. 1914), both wirtten in Tamil and a letter of Ramanujan to C.N. Ganapathy Iyer, dated 17 Dec. 1914.

Setting up a Science Museum, where emphasis is not on the Technological Applications but on the underlying scientific facts and their discovery requires ingenuity, skill and planning, by the best brains of the country. One can take the examples of similar museums in the world: the Deutsches Museum, Münich; the Science Museum, London and the Museum for Pi in Paris, and in India, museums established at Bangalore, Calcutta and Pilani.

If and when our Government establishes such a National Science Museum, to popularize and perpetuate the contributions of our Scientists (as much as it exhibits time and again its concern for its politicians), and our scientists and politicians take pride to preserve our heritage, then our Government can approach the British Government for the original papers of the *Natural Genius* Ramanujan. In such a National Science Museum, the works of our great scientists – Sir C.V. Raman, Satyendra Nath Bose, S. Chandrasekar – can be preserved for posterity.

Awareness today

One hundred and ten years after the birth of Ramanujan, it is no exaggeration to say that the interest in Ramanujan is still alive and unabated not only in the world of mathematics but also in the the world of the media. One may ask: how ? As has been stated earlier on, as long as people do mathematics Ramanujan's work will have a pride of place through publications of researchers in international journals of repute. There are, in fact, three journals named after Ramanujan: *The Hardy–Ramanujan Journal*, edited by Profs. R. Balasubramanian and K. Ramachandra and publisheded by Prof. K. Ramachandra of the Tata Institute of Fundamental Research, Bombay, on behalf of the Hardy-Ramanujan Society; the *Journal of the Ramanujan Mathematical Society*, edited by Prof. V. Kumar Murty and published by the Ramanujan Mathematical Society, started in 1985 with the initiative of Prof. R. Balakrishnan and a few others and *The Ramanujan Journal*, started in 1997 with a galaxy of 27 mathematicians and Prof. Krishnaswami Alladi as its Editor-in-Chief – An International Journal Devoted to the Areas of Mathematics Influenced by Ramanujan – and published by Kluwer Academic Publishers. This is a tribute befitting the great. Also in popular magazines there are articles – for instance in the March 1998 issue

of *Resonance* there is an article entitled *Ramanujan and I* by the renowned Hungarian mathematician (late) Prof. Paul Erdös[3], hailed as the *Western Ramanujan*. However, I have a tale to tell regarding what happens in the media world!

In the Saturday editions of two of the reputed English newspapers of India, on March 7, 1998, two articles appeared. One of these was a well written article about the Ramanujan Museum in Royapuram, the realization of the pioneering effort of Mr. P.K. Srinivasan. In the other, it was an article[4] entitled: *The Ramanujan factor* with an eye-catching photograph in the center of the article. But, alas! The photo was not that of Ramanujan! Needless to say, in the next week's magazine section the newspaper carried a brief regret for the bloomer. Instead of brushing this aside as just another journalistic error, I would like to ask: Would this have happened if there is public awareness of the portrait of Ramanujan ? If not, how can this be achieved ?

Most countries have the tradition of honouring their great scientists also by bringing out special commemorative Stamps and First Day Covers on significant days. The March 1998 issue of the *Notices of the American Mathematical Society* (Vol. **45**, No. 3) carries on its cover *A Collection of Mathematical Stamps* – of Ramanujan, Bolzano, Lagrange, Pascal, Kovalevskaya, al-Khuwarizmi, Dedekind, Newton and Cauchy. The Indian Philately Association has been ever vigilant on this count, and on the occasion of the 75th

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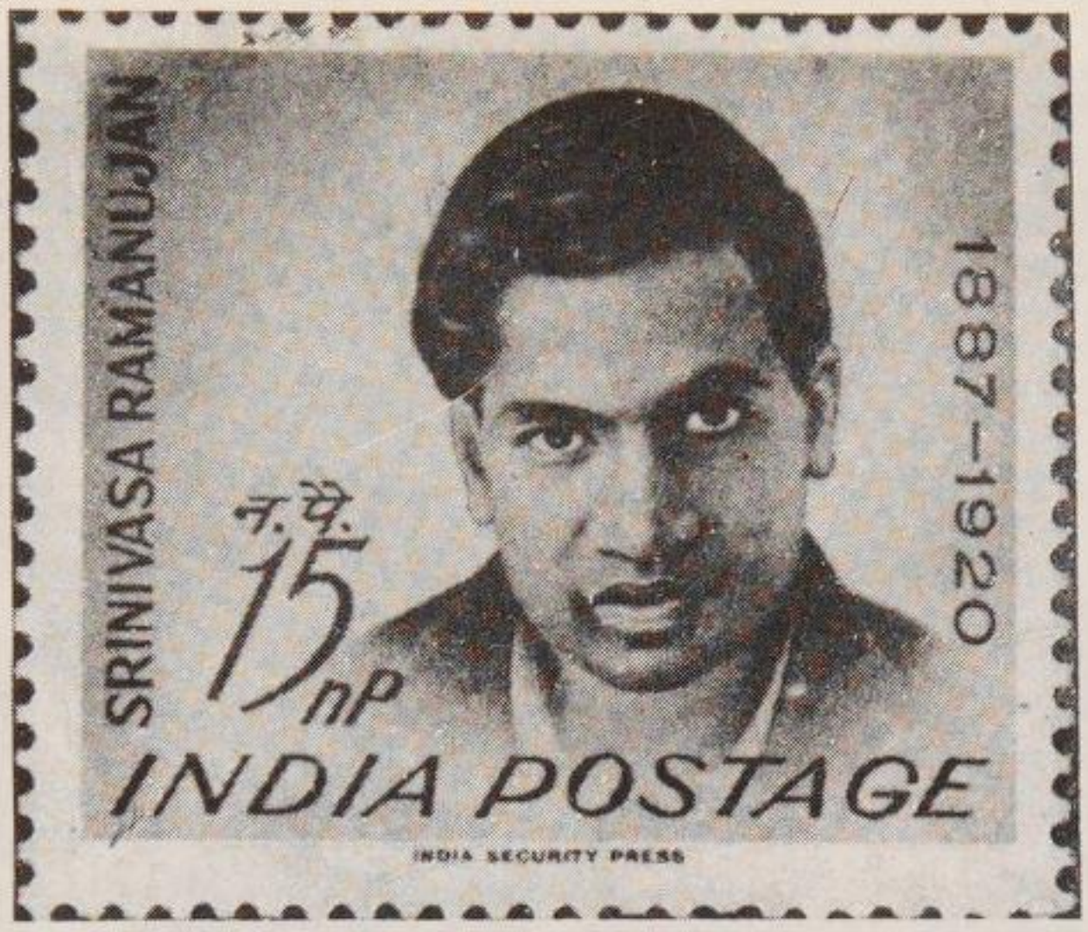
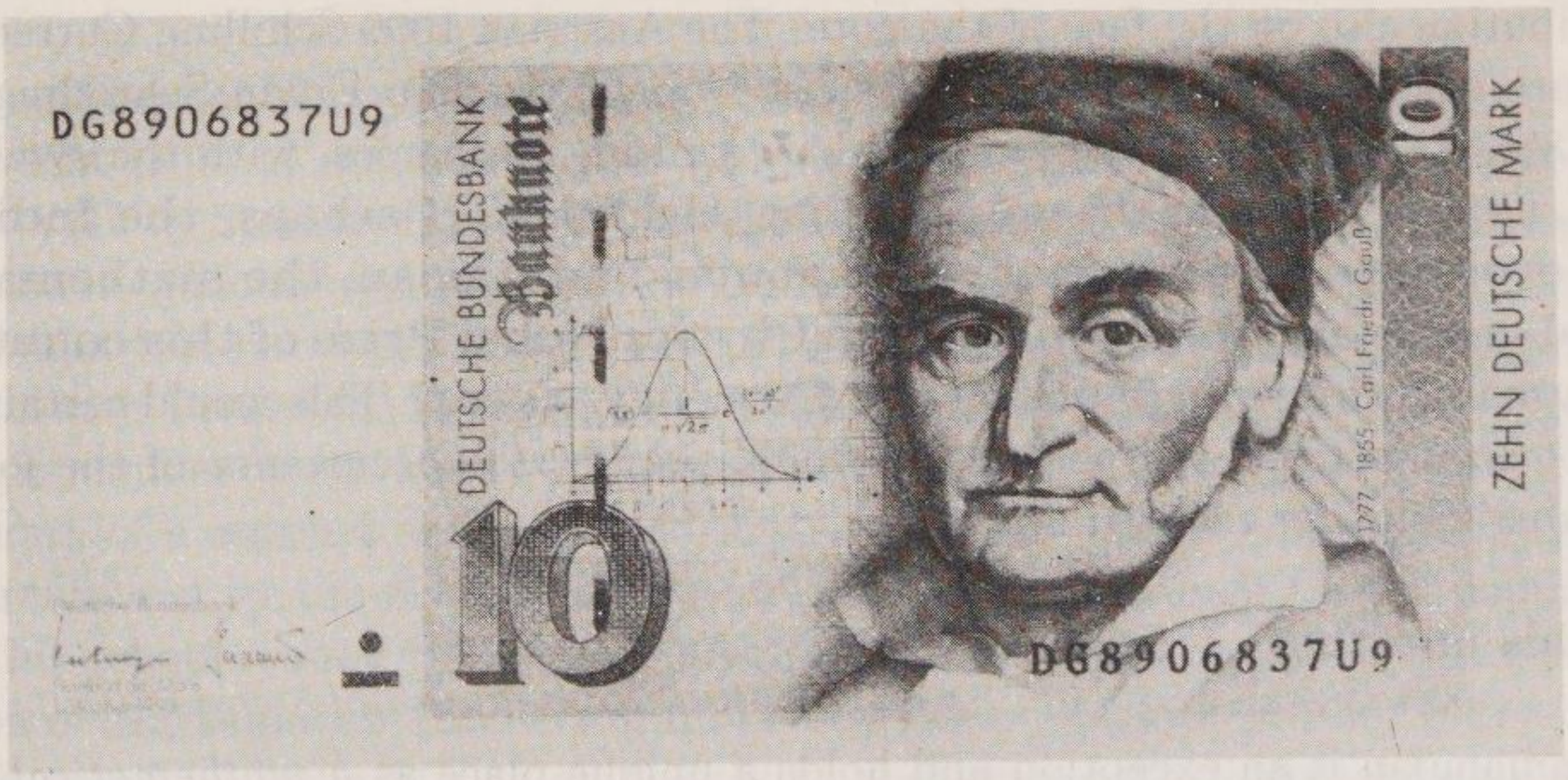
[3]The article is reproduced from [92]. *Resonance* also carried in its Dec. 1996 issue an article entitled: *Reflections Around the Ramanujan Centenary*, by the Fields Medalist Atle Selberg who had participated in the International Conference held in Madras in Dec. 1987 and in the Number Theory Conference which followed at Kandy in Sri Lanka.

[4]Coincidentally, I am aware of the genesis of this article. My friend Mr. C.A. Reddi was requested to enlarge on the *Ramanujan Family Record* supposed to have been compiled by S. Lakshmi Narasimhan, a younger brother of Ramanujan, in 40 pages but for which only scanty information is available (as reported in [XI], p. 314). Unable to locate the originals with the relatives of Ramanujan, Mr. Reddi wrote a 11-page article, some time in June 1997, entitled: *The Early and Last Days of Srinivasa Ramanujan*. Mr. Reddi had a shocking surprise when he saw the article in its much abridged form under a new title and accompanied by the photo of one Mr. S. Rangaswami and not Ramanujan!



Birth Anniversary of Ramanujan a commemorative stamp was brought out by the Government of India. While there exists a *Stamp Corner* column in the *Mathematics Intelligencer*, the author is unaware of one such for Currency Notes! A few countries[5] also honour their great scientists by using their portraits on Currency Notes. Two such examples are shown here:



Carl Friedrich Gauss (1777 – 1855).
The German 10 Deutsch Mark Currency.

[5]The author has not investigated this aspect but for a few enquiries and would appreciate receiving information on this aspect from the reader.

Erwin Schrödinger (1887 – 1961).
The Austrian 1000 Schilling Currency.

The German 10 Deutsch Mark Currency Note carries the portrait of the renowned Carl Friedrich Gauss (1777 – 1855) with the ubiquitous Gauss distribution also on the face of the note. The Austrian 1000 Schilling Currency Note has the portrait of the Physicist Nobel Prizeman Erwin Schrödinger, (1887 – 1961) one of the founders of Quantum Mechanics, with the symbol Ψ, used to represent the wave function, highlighted. **Perhaps, the Indian Government could honour Ramanujan and Raman, the mathematical genius and the winner of the Physics Nobel Prize of this country by using their portraits on the Currency Notes!** This would certainly not go unnoticed and perhaps, it would prevent the occurrence of the journalistic blunder referred above.

This and that

Ramanujan is an exception and holds a unique place as *a mathematical genius*. India has no dearth of students with talent. Due to the fact that no laboratory is required for starting post-graduate courses, most colleges in India start with a Master's degree programme in Mathematics. A large number of students enroll for these courses too. The author came across two outstanding students interested in Number Theory, during his continuous associ-

ation with the prestigious Institute of Mathematical Sciences ('Matscience'), Madras, as a student from 1964 and a faculty member from 1972. One of them was about nine years old in 1964 and got over his initial infatuation for aviation as a boy to take to mathematics. At college, he got interested in Fibonacci numbers and Farey series. He came into close contact with the prodigious Paul Erdös – renowned for his path breaking works in probabilistic number theory, combinatorics and graph theory – on his first visit to the Institute (and to India) in 1974. The young Alladi Krishnaswami, after completing his B.Sc. degree in Mathematics at the Vivekananda College, went to University of California at Los Angeles and within three years completed his Ph.D. under the Supervision of Prof. E.G. Strauss. After a few years as a post-doctoral fellow at the University of Michigan, Ann Arbor, he was for some years (1981 – 1990) a faculty member at 'Matscience', before he migrated to the United States were he soon rose to become a Professor at the Department of Mathematics of the University of Florida, Gainesville. Prof. Krishnaswami Alladi has since established his reputation for work in many branches of number theory and collaborated with Profs. P. Erdös, George E. Andrews and D. Bressoud. His rising stature enabled him to start *The Ramanujan Journal* published by Kluwer, from 1997.

Another promising young man, K. Sounderarajan, spotted as a precocious student [100] of mathematics was brought to 'Matscience' when he was in his VIII th Standard, in 1985, by the Mathematics teachers of the Padma Seshadri School. His regular visits to Matscience and its excellent Library brought him into close contact with Prof. R. Balasubramanian[6] and through him with Prof. C.S. Seshadri, F.R.S. He found, in 1985, that the only four digit number which has two repeated adjacent digits and which is itself the square of a number with repeated digits is: $7744 = 88^2$. Encouraged by his parents and by the School, Sound – the nickname by which he was popular in his school – took the annual Olympiad Tests conducted by the Association of Mathematics Teachers of India (AMTI), consecutively from 1987 – 91 and stood all-India first in all the tests. Spotted by the National Board for Higher Mathematics (NBHM) of the Department of Atomic energy, Govern-

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. [6]Prof. R. Balasubramanian is a Senior Professor now at 'Matscience' and well-known for his solution of the 200 year old Waring's Problem of 4th powers, along with the French Mathematicians, Profs. J.M. Deshoulles and F. Dress.

ment of India, he participated in their training camps and took up projects on Gödel's Incompleteness Theorem and Non-standard analysis. He was the reciepient of the award for the most outstanding student in Maths Expos for 1989–90. In the summer of 1989, he was one of the ten students chosen to participate in a six-week programme as a Research Scholar in the Research Science Institute (RSI) sponsored by the Center for Excellence in Education, Virginia, U.S.A., along with 58 American students and a few from Australia, China, France, Italy and Sweden. His paper *On the behaviour of a sparse series* was adjudged as one of the five best written projects in the RSI. His outstanding performance in the Scholastic Aptitude Test (SAT)[7] got him admission to the University of Michigan to pursue his under graduate studies. Prof. Hugh Montgomery of the University of Michigan made the required provisions to financially support Sound to join the University after his schooling in Madras. He has since successfully completed his under graduate studies at Michigan, published more than half a dozen publications in International journals, collaborated with Prof. R. Balasubramanian on a summer visit to Matscience, came into contact with several number theorists like: Profs. P. Erdös, Andrew Granville, L.C. Washington, A. Ivic, and is well on his way to completing his Doctoral thesis at University of Princeton, Princeton, under the Supervision of Prof. Peter Sarnack. He is considered as one of the brightest Analytic Number Theorists of recent times.

These are two success stories of youngsters spotted as precocious students of Mathematics, from independent India, whose recognition started from within the country leading naturally to international recognition. With modern day communications being excellent – especially, in research institutions like the TIFR, I.I.Sc., Matscience – we can safely state that the difficulties faced by Srinivasa Ramanujan, at the beginning of this century, regarding recognition, funding or finding (vegetarian) food abroad, are a thing of the past. However, there is no scope to be complacent and constant vigil must be there to spot and nurture talent. Only then can we hope to spot Soundararajans, and set the goals very high to achieve excellence in mathematics and hope that a miracle like a Ramanujan would appear, from the blue!

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[7]His score of 770 in the SAT led to his being ranked as *one of the most outstanding international students* of the 71 who had attended since 1984.

Video programmes

During Ramanujan's birth centenary year, 1987, a two-part Video programme on *Letters from an Indian Clerk* was made by *An Independent Communications Association Production for Cannel 4 with WGBH/Boston.* In it Ramanujan's Kumbakonam and Hardy's Cambridge are shown with interviews of Profs. Bela Bollobas, a Hungarian Number Theorist at Cambridge; George E. Andrews, the discoverer of the *Lost* notebook; S. Chandrasekhar, the Astrophysicist who was an admirer of Ramanujan all his life; and C.J. Hamson, Emeritus Professor of Law at the University of Cambrdige, who was then a surviving contemporary of Hardy and Littlewood. This tastefully made documentary (about 55 minutes duration) was produced and directed by the renowned Christopher Sykes. The visuals of Ramanujan were his passport photograph, the photo of his taken after his graduation (wearing a cap and gown) with other graduates, the bust of his made by Paul Granlund. Mrs. Janakiammal was also interviewed and she recalls how Ramanujan was always interested in writing down his 'sums' on paper; that mathematics was everything for him in this universe; and that food was given to him by her by hand while he was writing down his 'sums'; that on his death bed he assured her that whether he is alive or dead, his mathematics would provide for her. It is these pages which he was writing down furiously as he was bed ridden and dying which constitute his *Lost* notebook, which is considered by mathematicians today as perhaps the best work Ramanujan did.

In India, a 30-minute video programme was made entitled: *The enigma of Srinivasa Ramanujan* by the *National Council for science and Technology Communications* and credits are given to Drs. B.D. Acharya and N.K. Sehgal of the Department of Science and Technology, Government of India and to the following mathematicians: Drs. M.S. Rangachari, S. Raghavan, S.S. Rangachari, K. Ramachandra and to Mrs. Janakiammal and her foster son W. Narayanan. In this short documentary, the role of Ramanujan is played by a child, a young man and an adult (Raghubir Yadav) while Hardy is portrayed by Tom Alter. Hardy's words that Ramanujan is neither an 'inspired idiot' nor a 'psychological creep' but a rational human being are mouthed by the actor. The temple town of Kumbakonam and the Town High School where Ramanujan studied are shown, besides the goddess Na-

magiri of Namakkal. In typical filmy style, for a brief moment the young Ramanujan is shown running around the pillars with the little girl Janaki.

Web sites

For those who wish to look up a web site for References for Srinivasa Ramanujan, look up:

http://www-groups.dcs.st-and.ac.uk/~history/Mathematicians

The web sites of Profs. Richard Askey, Bruce C. Berndt and the author are:

http://www.math.wisc.edu/~askey
http://www.math.uiuc.edu/~berndt
http://www.imsc.ernet.in/~rao

The home page of Prof. Askey contains a link to Srinivasa Ramanujan and clicking on it shows the photo of the bust of Ramanujan sculpted by Paul Granlund. The home page of Prof. Bruce Berndt has links to his book publications, viz. Parts I to V of the *Ramanujan Notebooks* and the book *Ramanujan: Letters and Commentary* (ed. by Berndt and Rankin), besides a link to *the Ramanujan Journal* for which he is an Editor.

Concluding remarks

In this book an attempt has been made to string together the facts as we know them today from the published accounts of Ramanujan's contemporaries and biographers. The work of Ramanujan is permanently enshrined in the *Collected Papers* [III] and the Notebooks of Ramanujan [VII]. The 12 lectures in *Ramanujan* by Hardy [IV] and the five volume work of Bruce Berndt [VIII] entitled the *Ramanujan Notebooks*, besides innumerable articles and research publications are the outcome of the brilliant work of Ramanujan. The scope and objects of this book were restricted to provide a first introduction of his life and work and to convey enough to arouse the interest of the teachers and students of mathematics in schools and colleges and to point to the original sources for further study. It is the author's hope that this book would inspire some to seek the originals in well equipped libraries and lead a few to follow the researches on Ramanujan's inspired insights to discover new results too! Just as an example of what is the premise for such a hope I wish to state that a comment of Hardy in the notes at the end of a lecture of his

on hypergeometric series in [IV] triggered a line of thought and discussions on it (during Oct.–Nov. 1997, at the University of Ghent, Belgium) with Prof. J. Van der Jeugt resulted in our writing an interesting article on the group theory of transformations of basic hypergeometric series. On this optimistic note the author urges the inspired mathematics student to start his wanderings through *Ramanujan's garden* and start locating beautiful floral results that catch the beholder's eye.

Appendix 1. Research publications of Ramanujan

Papers written by S. Ramanujan:

1. Some Properties of Bernoulli's numbers
   Journal of the Indian Mathematical Society, **3** (1911) 219–234.

2. On Question 330 of Prof. Sanjana
   Journal of the Indian Mathematical Society, **4** (1912) 59–61.

3. Note on a set of simultaneous equations
   Journal of the Indian Mathematical Society, **4** (1913) 94–96.

4. Irregular numbers
   Journal of the Indian Mathematical Society, **5** (1913) 105–106.

5. Squaring the circle
   Journal of the Indian Mathematical Society, **5** (1913),132.

6. Modular equations and approximations to π
   Quarterly Journal of Mathematics, **45** (1914) 350–372.

7. On the integral $\int_0^x \frac{tan^{-1}t}{t} dt$
   Journal of the Indian Mathematical Society, **7** (1915) 93–96.

8. On the number of divisors of a number
   Journal of the Indian Mathematical Society, **7** (1915) 131–133.

9. On the sum of the square roots of the first n natural numbers
   Journal of the Indian Mathematical Society, **7** (1915) 173–175.

10. On the product $\prod_{n=0}^{n=\infty} \left[1 + \left(\frac{x}{a+nd}\right)^3\right]$
    Journal of the Indian Mathematical Society, **7** (1915) 209–211.

11. Some definite integrals
    Messenger of Mathematics, **44** (1915) 10–18.

12. Some definite integrals connected with Gauss's sums
    Messenger of Mathematics, **44** (1915) 75–85.

13. Summation of certain series
Messenger of Mathematics, **44** (1915) 157–160.

14. New expressions for Riemann's functions $\xi(s)$ and $\Xi(t)$
Quarterly Journal of Mathematics, **46** (1915) 253–260.

15. Highly composite numbers
Proceedings of the London Mathematical Society, **2**, 14 (1915) 347–409.

16. On certain infinite series
Messenger of Mathematics, **45** (1916) 11–15.

17. Some formulae in the analytic theory of numbers
Messenger of Mathematics, **45** (1916) 81–84.

18. On certain arithmetical functions
Transactions of the Cambridge Philosophical Society, **22**, No. 9 (1916) 159–184.

19. A series for Euler's constant γ
Messenger of Mathematics, **46** (1917) 73–80.

20. On the expression of a number in the form $ax^2 + by^2 + cz^2 + du^2$
Proceedings of the Cambridge Philosophical Society, **19** (1917) 11–21.

21. On certain trigonometrical sums and their applications in the theory of numbers
Transactions of the Cambridge Philosophical Society, **22**, No. 13 (1918) 259–276.

22. Some definite integrals
Proceedings of the London Mathematical Society, **2**, 17 (1918) Records for 17 Jan. 1918.

23. Some definite integrals
Journal of the Indian Mathematical Society, **11** (1919) 81–87.

24. A proof of Bertrand's postulate
Journal of the Indian Mathematical Society, **11** (1919) 181–182.

25. Some properties of $p(n)$, the number of partitions of n
Proceedings of the Cambridge Philosophical Society, **19** (1919) 207–210.

26. Proof of certain identities in combinatory analysis
Proceedings of the Cambridge Philosophical Society, **19** (1919) 214–216.

27. A class of definite integrals
Quarterly Journal of Mathematics, **48** (1920) 294–310.

28. Congruence properties of partitions
Proceedings of the London Mathematical Society, **2**, 18 (1920) Records for 13 March 1919.

29. Algebraic relations between certain infinite products
Proceedings of the London Mathematical Society, **2**, 18 (1920) Records for 13 March 1919.

30. Congruence properties of partitions
Mathematische Zeitschrift, **9** (1921) 147–153.

Papers written by S. Ramanujan with G.H. Hardy

- Une formulae asymptotique pour le nombre des partitions de n
Comptès Rendus, 2 Jan. 1917.

- Proof that almost all numbers n are composed of about log log n prime factors
Proceedings of the London Mathematical Society, **2**, 16 (1917) Records for 14 Dec. 1916.

- Asymptotic formulæin combinatory analysis
Proceedings of the London Mathematical Society, **2**, 16 (1917) Records for 1 March 1917.

- Asymptotic formulae for the distribution of integers of various types
Proceedings of the London Mathematical Society, **2**, 16 (1917) 112–132.

- The normal number of prime factors of a number n
Quarterly Journal of Mathematics, **48** (1917) 76–92.

- Asymptotic formulae in combinatory analysis
  Proceedings of the London Mathematical Society, **2**, 17 (1918) 75–115.

- On the coefficients in the expansions of certain modular functions
  Proceedings of the Royal Society, A, **95** (1918) 144–155.

These 37 research publications of Ramanujan are reprinted in the *Collected Papers by Srinivasa Ramanujan*, edited by G.H. Hardy, P.V. Seshu Aiyar and B.M. Wilson and published first by the Cambridge University Press (1927) and later by Chelsea (1962). The Questions posed and the Answers provided by Ramanujan, 59 in all, in the Journal of the Indian Mathematical Society follow the research papers in the *Collected Papers*. Besides the Preface, the editors include two biographical *Notices*, one by P.V. Seshu Aiyar and R. Ramachandra Rao and another by G.H. Hardy, at the beginning of the *Collected Papers*. Notes on the research publications and some extracts from Ramanujan's letters to G.H. Hardy are in two Appendices, at the end of the *Collected Papers*.

Appendix 2. Wren Library Card Catalogue and Papers of Ramanujan

With the help of Professor R.A.Rankin, the Wren Library has catalogued the papers of Ramanujan. These have the reference addresses as Add.Ms.a.94, with a superscript which refers to the specific document, and as Add.Ms.b.100 – 107C.

The papers that are available with reference Add.Ms.a.94^{1-18}, are contained in a large sized (A3-size) 2 in. high cardboard box. The references Add.Ms.b. 100 - 107C refer to the five bound volumes (fullscape size) and, in addition to these, the papers and letters are preserved in two large (A3) sized thick card board boxes.

In this Appendix are given the details of the Card Catalogue on Ramanujan, as well as the details of the Ramanujan MSS as maintained in the Archives of the Wren Library of Trinity College. In fact, a part of Appendix Card Catalogue is what is contained in Supplement II of Venkatachaliengar's article[6], as "A list of all the Ramanujan-material at the Trinity College Cambridge ... given to Professor C.T.Rajagopal by Rankin".

The Card Catalogue in the Wren Library of Trinity College:

1. AIYAR (P Venkatesvara Seshu)
   see AYYAR (P Venkatesvara Seshu)

2. ANDERSON (Sir Hugh Kerr)
   see HARDY (G.H.)

3. AYYAR (P Venkatesvara Seshu)
   Letter to G.H.Hardy, 23 March 1928
   Add.Ms.a.$94^{1(31)}$

4. AYYAR (P Venkatesvara Seshu)
   Letter to B.M.Wilson, 8 July 1925
   Add.Ms.a.$94^{12(49)}$

5. AYYAR (P Venkatesvara Seshu)
   Letter to B.M.Wilson, 22 April 1926
   Add.Ms.a.94$^{12(58)}$

6. BERWICK (William Edward Hodgson)
   Letter [to G.N.Watson ?], 24 Oct. 1916
   Add.Ms.a.94^{5}

7. CAMBRIDGE UNIVERSITY PRESS
   see HARDY (G.H.)
   RAMANUJAN (S.)

8. CHANDRASEKHAR (Subrahmanyan)
   Letter to G.H.Hardy, 4 Aug. 1937 [and 28 Dec. 1937]
   Add.Ms.a.94^{8}

9. CLARK (Edward Mellish)
   Letters to G.H.Hardy, 7 - 25 May 1920
   Add.Ms.a.94$^{1(21-24)}$

10. DEWSBURY (Francis)
    Letters to G.H.Hardy, 5 March 1918 - 15 Jan. 1929
    Add.Ms.a.94$^{1(13-17)}$

11. DEWSBURY (Francis)
    Letter to G.H.Hardy, 13 July 1922
    Add.Ms.a.94$^{12(40)}$

12. DEWSBURY (Francis)
    Letter to G.H.Hardy, 2 Aug. 1923
    Add.Ms.a.94$^{12(25)}$

13. DEWSBURY (Francis)
    Letter to G.H.Hardy, 6 Aug. - 13 Sept. 1923
    Add.Ms.a.94$^{(27-29)}$

14. FOWLER (Sir Ralph Howard)
    Letter to G.H.Hardy, 9 June [1922]
    Add.Ms.a.94$^{12(34)}$

15. HARDY (Godfrey Harold)
"G.H.Hardy in account with the Syndics of the Cambridge University press : Collected Papers of S.Ramanujan. Statement of Account from 1 January 1928 to 31 December 1928", April 1929.
Add.Ms.a.94$^{12(77)}$

16. HARDY (Godfrey Harold)
Letter to S.Ramanujan, 6 Feb. 1918, on summing 'partition series' for $\theta_3^r(\tau)(r \ \ odd)$
Add.Ms.a.94^{16}

17. HARDY (Godfrey Harold)
Letter to G.N.Watson, [April 1929]
Add.Ms.a.94$^{12(76)}$

18. HARDY (Godfrey Harold)
Letter to G.N.Watson, [n.d.[1]]
Add.Ms.a.94^{3}

19. HARDY (Godfrey Harold)
Letter to B.M.Wilson, [5 June 1925]
Add.Ms.a.94$^{12(32)}$

20. HARDY (Godfrey Harold)
Letter to B.M.Wilson, [10 June 1925]
Add.Ms.a.94$^{12(41-43)}$

21. HARDY (Godfrey Harold)
Envelope addressed to B.M.Wilson, 14 oct. 1925
Add.Ms.a.94$^{12(56)}$

22. HARDY (Godfrey Harold)
Letter to B.M.Wilson, 15 Oct. [1925]
Add.Ms.a.94$^{12(52)}$

23. HARDY (Godfrey Harold)
Letter to B.M.Wilson, [Jan./Feb. 1927]
Add.Ms.a.94$^{12(62)}$

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[1]n.d. stands for not dated

24. HARDY (Godfrey Harold)
Letters to B.M.Wilson, [n.d.]
Add.Ms.a.94$^{12(5-8)}$

25. HARDY (Godfrey Harold)
Letters to B.M.Wilson, [n.d.]
Add.Ms.a.94$^{12(20-21)}$

26. RAMANUJAN (Srinivasa)
Letter on mock theta functions. [This appears to be the letter of 12 January 1920, partly reproduced on pp.354 - 355 of the Collected Papers.]

27. HARDY (Godfrey Harold)
"Memorandum of agreement made this 12 August 1925 between Professor G.H.Hardy, New College, Oxford ... and the Syndics of the University Press Cambridge ..." [concerning S.Ramanujan's Collected Papers], signed by G.H.Hardy and Sir H.K.Anderson.
Add.Ms.a.94$^{12(53)}$

28. HARDY (Godfrey Harold)
'Mr.S.Ramanujan's Mathematical Work in England : Report by Mr.G.H.Hardy to the University of Madras' (pr.), Cambridge 1916.
Add.Ms.a.94$^{12(79)}$

29. HARDY (Godfrey Harold)
Work on hypergeometric series and summation of series of powers of binomial coefficients, [n.d.]
Add.Ms.a.94$^{(17)}$

30. HARDY (Godfrey Harold)

see AYYAR(P.V.S.)
CHANDRASEKHAR (S.)
CLARK (E.M.)
DEWSBURY (F.)
FOWLER (Sir)
HENSMAN (J.J.)
JEANS (Sir)
KINGSFORD (R.J.L.)
KOMALATAMMAL
LEHMER (D.H.)
LITTLEWOOD (J.E.)
MENON (K.R.)
NARASIMHAN (S.L.)
RAMALINGAM (A.S.)

31. HARDY (Godfrey Harold)
see RAO (K.A.)
RAO (R.R.)
ROBERTS (Sir S.C.)
SPRING (Sir F.J.E.)
WALLER (A.R.)

32. HEILBRONN (Hans Arnold)
see WATSON (G.N.)

33. HENSMAN (John J) Letter to G.H.Hardy, 29 April 1920
Add.Ms.a.94[1(18)]

34. JEANS (Sir James Hopwood)
Letter to G.H.Hardy, 15 June 1922
Add.Ms.a.94[12(33)]

35. KINGSFORD (Reginald John Lethbridge)
Letter to G.H.Hardy, 21 August 1925
Add.Ms.a.94[12(51)]

36. KINGSFORD (Reginald John Lethbridge)
Letter to G.H.Hardy, 23 Aug. 1929
Add.Ms.a.94[12(78)]

37. KOMALATAMMAL
Letter to G.H.Hardy, 25 Aug. 1927
Add.Ms.a.94[1(27–28)]

38. LEHMER (Derrick Henry)
Letter to G.H.Hardy, 2 Nov. 1937
Add.Ms.a.94[6(1–2)]

39. LITTLEWOOD (John Edensor)
Letter to G.H.Hardy, [1913 ?]
Add.Ms.a.94[1(2–5)]

40. LITTLEWOOD (John Edensor)
Note [to G.N.Watson], [June 1948]
Add.Ms.a.94[1(1)]

41. LITTLEWOOD (John Edensor)
Card to B.M.Wilson, 4 March [1927]
Add.Ms.a.94[12(67)]

42. McLEAN (William)
Letter to B.M.Wilson, 13 Aug. 1925
Add.Ms.a.94[12(50)]

43. McLEAN (William)
Letter [copy] to B.M.Wilson, 13 Aug. 1925
Add.Ms.a.94[12(54)]

44. McLEAN (William)
Letter to B.M.Wilson, 31 March 1927
Add.Ms.a.94[12(68)]

45. MENON (K Ramunni)
Letter [copy] to G.H.Hardy, 28 July 1920
Add.Ms.a.94[1(25)]

46. NARASIMHAN (S Lakshmi)
Letter [to G.H.Hardy], 29 April 1920
Add.Ms.a.94[1(19)]

47. NEVILLE (Eric Harold)
Card to B.M.Wilson, 28 Dec. 1927
Add.Ms.a.94[12(70)]

48. NEVILLE (Eric Harold)
'The Late Srinivasa Ramanujan', letter pr. in Nature, No.2673, 20 Jan. 1921, pp. 661 - 662.
Add.Ms.a.94[12(71)]

49. OPPENHEIM (Tan Sri Sir Alexander)
Letter to B.M.Wilson, 31 Dec. 1926
Add.Ms.a.94[12(60)]

50. RAM (A S)
see RAMALINGAM (A S)

51. RAMALINGAM (A S)
Letter to G.H.Hardy, 23 June 1918.
Add.Ms.a.$94^{1(7-9)}$

52. RAMALINGAM (A S)
Letter to S.Ramanujan, 23 June 1918.
Add.Ms.a.$94^{1(10-12)}$

53. RAMALINGAM (A S)
see RAMANUJAN (S.)

54. RAMANUJAN (Srinivasa)
Calculations 'copied from the loose papers' by an unknown hand.
Add.Ms.b.104

55. RAMANUJAN (Srinivasa)
Correspondence of G.H.Hardy with Cambridge University Press and others about S.Ramanujan's Collected Papers, 3 Dec. 1920 - 23, Aug. 1929 and [n.d.]
Add.Ms.a.94^{12}

56. RAMANUJAN (Srinivasa)
Displayed formulae for preperation as slides, [n.d.]
Add.Ms.a.94^{9}

57. RAMANUJAN (Srinivasa)
Enlarged passport photograph, [1913 ?] [5 copies]
Add.Ms.a.94^{7}

58. RAMANUJAN (Srinivasa)
Letters to G.H.Hardy, 28 June 1918 and [n.d.]
Add.Ms.a.94^{2}

59. RAMANUJAN (Srinivasa)
Letter to A.S.Ramalingam, 19 June 1918
Add.Ms.a.$94^{1(16)}$

60. RAMANUJAN (Srinivasa)
Letter [incomplete], [n.d.]
Add.Ms.a.$94^{6(3-8)}$

61. RAMANUJAN (Srinivasa)
List of suggested changes in the biographical sketch of S.Ramanujan [for S.Ramanujan's Collected Papers], [1923].
Add.Ms.a.94[12(30)]

62. RAMANUJAN (Srinivasa)
' 'Lost' notebook' on $q-$ series and similar topics, [n.d.]
Add.Ms.a.94[15]

63. RAMANUJAN (Srinivasa)
'Minutes of the Congratulatory Meeting held in honour of Mr.S. Ramanujam and Mr.K.Ananda Rao' [copy], 22 March 1928
Add.Ms.a.94[1(29–30)]

64. RAMANUJAN (Srinivasa)
Miscellaneous mss : highly composite numbers, singular moduli, Rogers-Ramanujan identities, Mersenne numbers, etc. with annotations by G.K.Stanley and G.H.Hardy, and a letter from G.H.Hardy to G.N.Watson, [1930].
Add.Ms.a.94[18]

65. RAMANUJAN (Srinivasa)
'Mr.Ramanujan's accounts' [copy], 28 Nov. 1917 - 23 Aug. 1919
Add.Ms.a.94[1(20)]

66. RAMANUJAN (Srinivasa)
'Notebook 2', copy by an unknown hand. 4 Vols.
Add.Ms.b.101 - 104

67. RAMANUJAN (Srinivasa)
'Notes on the biographical sketch by Mr.S.Ramanujam, F.R.S.' [copy] [for S.Ramanujan's Collected Papers], [n.d.]
Add.Ms.a.94[12(26)]

68. RAMANUJAN (Srinivasa)
Papers Add.Ms.a.94
Add.Ms.b.100 - 107C

69. RAMANUJAN (Srinivasa)
'Properties of $p(n)$ and $\tau(n)$'. [This is the paper of which part is published in Collected Papers, No.30 (pp.232 - 238), under the title 'Congruence Properties of Partitions'] [n.d.]
Add.Ms.a.94[13]

70. RAMANUJAN (Srinivasa)
'Report on the proposed publication of the Collected Mathematical Works of S.Ramanujan' [copy] with annotations by B.M.Wilson. [n.d.]
Add.Ms.a.94[12(44–46)]

71. RAMANUJAN (Srinivasa)
Reminiscences of, by P.Adinarayana Chetty, copied from the Madras Mail, 27 Apr. 1920
Add.Ms.a.94[3(18)]

72. RAMANUJAN (Srinivasa)
see NEVILLE (E.H.)
RAMALINGAM (A.S.)
SATAGOPAN (T.A.)
STORMER (F.C.M.)
WATSON (G.N.)
WILSON (B.M.)

73. RAO (R Ramachandra)
Letter [to G.H.Hardy], 3 Dec. 1920
Add.Ms.a.94[12(22)]

74. RAO (K Ananda)
Letter to G.H.Hardy, 26 May 1921
Add.Ms.a.94[12(23)]

75. RAO (K Ananda)
Letter to G.H.Hardy, 7 April 1936
Add.Ms.a.94[1(32)]

76. ROBERTS (Sir Sydney Castle)
Letters to G.H.Hardy, 30 Aug. [copy] - 23 Sept. 1922
Add.Ms.a.94[12(37–39)]

77. ROBERTS (Sir Sydney Castle)
Letter to G.H.Hardy, 10 April 1923.
Add.Ms.a.94[12(24)]

78. ROBERTS (Sir Sydney Castle)
Letter to G.H.Hardy, 6 June 1925.
Add.Ms.a.94[12(35)]

79. ROBERTS (Sir Sydney Castle)
Letter to G.H.Hardy, 8 Oct. 1925.
Add.Ms.a.94[12(55)]

80. ROBERTS (Sir Sydney Castle)
Letter to G.H.Hardy, 11 Nov. 1925.
Add.Ms.a.94[12(57)]

81. ROBERTS (Sir Sydney Castle)
Letter to G.H.Hardy, 7 Jan. 1926 [sic; i.e. 1927]
Add.Ms.a.94[12(63)]

82. ROBERTS (Sir Sydney Castle)
Letter [copy] to G.H.Hardy, 7 Jan. 1927
Add.Ms.a.94[12(65)]

83. ROBERTS (Sir Sydney Castle)
Letter to B.M.Wilson, 12 June 1925
Add.Ms.a.94[12(47)]

84. SATAGOPAN (T A)
Copy of S.Ramanujan's 'Notebook 1', [n.d.]
Add.Ms.b.100

85. SCOTT (Francis Reginald Fairfax)
Letter to B.M.Wilson, 22 Dec. 1926
Add.Ms.a.94[12(59)]

86. SCOTT (Francis Reginald Fairfax)
Letter to B.M.Wilson, 10 Jan. 1927
Add.Ms.a.94[12(61)]

87. SPRING (Sir Francis Joseph Edward)
Letter to G.H.Hardy, 18 Feb. 1921
Add.Ms.a.94[1(26)]

88. STØRMER (Carl)
see STØRMER (Fredrik Carl Mülertz)

89. STØRMER (Fredrik Carl Mülertz)
'Srinivasa [sic] Ramanujan - et merkelig matematisk geni', article pr. in Lördagsavisen, 6 Jan. 1934
Add.Ms.a.94[11]

90. TOWLE (F A)
Letter to B.M.Wilson, 11 Feb. 1927
Add.Ms.a.94[12(66)]

91. TOWLE (F A)
Letter to B.M.Wilson 1 July 1925
Add.Ms.a.94[12(48)]

92. VIJAYARAGHAVAN (T)
Letter to B.M.Wilson, 4 Jan. 1928
Add.Ms.a.94[12(72–75)]

93. WALLER (Alfred Rayney)
Letter to G.H.Hardy, 3 June 1922
Add.Ms.a.94[12(36)]

94. WATSON (George Neville)
Letter to H.A.Heilbronn, 19 March 1939
Add.Ms.a.94[4]

95. WATSON (George Neville)
Copy of S.Ramanujan's 'Notebook 2', Chapters XII - XXI, with Watson's proofs of Ramanujan's formulae [incomplete], [n.d.]
Add.Ms.b.105-107A

96. WATSON (George Neville)
Copy of S.Ramanujan's three Quarterly Reports to the Board of Studies

in Mathematics, 5 Aug. and 7 Nov. 1913 and 9 March 1914
Add.Ms.b.107A

97. WATSON (George Neville)
Notes on S.Ramanujan and his work, [n.d.]
Add.Ms.a.94[10]

98. WATSON (George Neville)
see BERWICK (W.E.H.) LITTLEWOOD (J.E.)
HARDY (G.H.) RAMANUJAN (S.)

99. WILSON (Bertram Martin)
Account of S.Ramanujan and his work, [n.d.]
Add.Ms.b.107C[1]

100. WILSON (Bertram Martin)'Comments on Appendix, pp. 225 - 273'
[of S.Ramanujan's Collected Papers], [n.d.]
Add.Ms.a.94[12(3–4)]

101. WILSON (Bertram Martin)
'Comments on pp.1 - 16' [of S.Ramanujan's Collected Papers], [n.d.]
Add.Ms.a.94[12(1–2)]

102. WILSON (Bertram Martin)
General account of S.Ramanujan's life and work, [n.d.]
Add.Ms.b.107C[11]

103. WILSON (Bertram Martin)
Investigations on Chapters II - VII of S.Ramanujan's 'Notebook 2',
[n.d.]
Add.Ms.b.107B

104. WILSON (Bertram Martin)
Letter [copy] (to the guarantors of S.Ramanujan's Collected Papers),
7 Feb. 1927
Add.Ms.a.94[12(64)]

105. WILSON (Bertram Martin)
List of corrections to Chapters I - X of S.Ramanujan's 'Notebook 2',

with references to published work of other mathematicians, [n.d.]
Add.Ms.b.107C[2]

106. WILSON (Bertram Martin)
Notes and proofs of results in Chapters II - XIII of S.Ramanujan's 'Notebook 2', [n.d.]
Add.Ms.b.107C[3–10,12]

107. WILSON (Bertram Martin)
Notes on the biographical sketch of S.Ramanujan [for S.Ramanujan's Collected Papers], [1923]
Add.Ms.a.94[12(31)]

108. WILSON (Bertram Martin)
'Solutions of questions proposed by other authors' [with reference to S.Ramanujan's Collected Papers], [n.d.]
Add.Ms.a.94[12(9–19)]

109. WILSON (Bertram Martin)
see AYYAR (P.V.S.)
HARDY (G.H.)
LITTLEWOOD (J.E.)
McLEAN (W.)
NEVILLE (E.H.)
OPPENHEIM(Tan Sri Sir A.)
ROBERTS (Sir S.C.)
SCOTT (F.R.F.)
TOWLE (F.A.)
VIJAYARAGHAVAN (T.)

110. RAMANUJAN (Srinivasa)
see ANDREWS (G.E.)

111. RAMANUJAN (Srinivasa)
see BERNDT (B.C.)

112. RAMANUJAN (Srinivasa)
see HARDY (G.H.) and RAMANUJAN (S.)

113. RAMANUJAN (Srinivasa)
see LITTLEWOOD (J.E.)

114. RAMANUJAN (Srinivasa)
see Newspaper cuttings

115. RAMANUJAN (Srinivasa)
see Newspaper cuttings : Hardy (G.H.)
Neville (E.H.)
Watson (G.N.)
Littlewood (J.E.)

116. RAMANUJAN (Srinivasa)
see RANKIN, R.A.

117. RAMANUJAN (Srinivasa)
see RAMANATHAN (K.G.)

In the Wren Library of Trinity College, Cambridge, the manuscripts of Ramanujan are kept in three large (foolscape size) card board boxes. Besides these three boxes, there are five large ruled bound volumes. Given below are the details of the contents of these Ramanujan Papers accessible with Add.Ms.a.94 and Add.Ms.b.100 - 107C :

Add.Ms.a.94

1. (1) J.E.Littlewood to [G.N.Watson], [June 1948]
(2 - 5) J.E.Littlewood to G.H.Hardy, [? 1913]
(6) S.Ramanujan to A.S.Ramalingam, 19 June 1918
(7 - 9) A.S.Ramalingam to G.H.Hardy, 23 June 1918
(10 - 12) A.S.Ramalingam to S.Ramanujan, 23 June 1918
(13 - 17) Francis Dewsbury to G.H.Hardy, 1918 - 20 (5 letters)
(18) John H.Hensman to G.H.Hardy, 29 April 1920
(19) S.L.Narasimhan to [G.H.Hardy], 29 April 1920
(20) "Mr.Ramanujan's Accounts" [copy], November 1917 - August 1919
(21 - 24) E.M.Clark to G.H.Hardy, May 1920 (4 letters)
(25) K.R.Menon to G.H.Hardy, 28 July 1920 [copy]
(26) Sir Francis Spring to G.H.Hardy, 18 February 1921
(27 -28) Komalatammal to G.H.Hardy, 25 August 1927
(29 -30) Minutes of the Congratulatory Meeting held in honour of Mr.S. Ramanujan and Mr.K.Ananda Rao, 22 March 1928 [copy]
(31) P.V.S.Ayyar to G.H.Hardy, 23 March 1928

(32) K.A.Rao to G.H.Hardy, 7 April 1936

2. S.Ramanujan to G.H.Hardy, 28 June 1918 and n.d. (6 letters of which 4 are from Matlock House and 2 from Fitzroy House)

3. G.H.Hardy to G.N.Watson, n.d. (2 letters)

4. G.N.Watson to H.A.Heilbronn, 19 March 1939

5. W.E.H.Berwick to [G.N.Watson], 24 October 1916

6. (1 - 2) D.H.Lehmer to G.H.Hardy, 2 November 1937 S.Ramanujan (incomplete, undated letter)

7. Enlarged passport photograph of S.Ramanujan (5 copies)

8. S.Chandrasekhar to G.H.Hardy, 4 August 1937 and 28 December 1937 [both regarding the Passport sized photograph of Ramanujan]

9. S.Ramanujan – Displayed formulae for preperation as slides, n.d.

10. Notes on Ramanujan and his work by G.N.Watson, n.d.

11. Srinivasa [sic] Ramanujan - et merklig matematisk geni article printed in Lördagsavisen by F.C.M.Størmer, 6 January 1934 [Norwegian newspaper]

12. Correspondence of G.H.Hardy with Cambridge University Press and others about S.Ramanujan's Collected Papers, December 1920 - August 1929

13. S.Ramanujan "Properties of $p(n)$ and $\tau(n)$" [This is the paper of which part is published in Collected Papers, No. 30 (pp.232 - 238), under the title "Congruence Properties of Partitions"] n.d.

14. Letter on mock theta functions. [This appears to be the letter of 12 January 1920, partly reproduced on pp. 354 - 355 of the Collected Papers]

15. S.Ramanujan : "Lost Notebook" on q-series and similar types, n.d.

16. G.H.Hardy to S.Ramanujan, 6 February 1918 on summing 'partition series' for $\theta_3^r(\tau)$ $(r\ odd)$

17. G.H.Hardy : Work on hypergeometric series and summation of series of powers of binomial coefficients, n.d.

18. S.Ramanujan : Miscellaneous mss including highly composite numbers, singular moduli, Rogers - Ramanujan identities, Mersenne numbers, etc. With annotations by [Miss] G.K.Stanley and G.H.Hardy, and a letter from Hardy to G.N.Watson, [1930]

Add.Ms.b.100. T.A.Satagopan : Copy of S.Ramanujan's "Notebook 1" n.d.[2]

This is Ramanujan's Notebook 1, copied by T.A.Satagopan. The original was published in facsimilie edition without any commentary by the Tata Institute of Fundamental Research, Bombay in 1957 and in 1987, Noarosa Publishing House has reprinted it. This Notebook has 350 pages and covers Chapters I to XVI of the original Notebook 1 of Ramanujan.

Add.Ms.b.101 - 104. Copy of Ramanujan's "Notebook 2" by an unknown hand, n.d.

This is the first of four notebooks into which Ramanujan's Notebook 2 was copied. As in the case of Notebook 1, the original was published in facsimilie edition by the T.I.F.R., in 1957 and subsequently reprinted by Narosa in 1987. Chapters I to X of the Notebook 2 of Ramanujan are copied in this notebook in 138 pages. Chapter X is continued in Add.Ms.b.102. Add.Ms.b.102,Add.Ms.b.103 and Add.Ms.b.104 are, respectively 138, 139 and 118 pages in length and Chapter XXI of Ramanujan's Notebook 2 is copied at the end of Add.Ms.b.103 and it is continued in Add.Ms.b.104, which also contains calculations copied from Ramanujan's "loose papers" by an unknown hand. In Add.Ms.b.102 can be found two letters of Watson to Wilson (the two Mathematicians who undertook the task of editing the original Notebooks of Ramanujan and divided the Notebook 2 into two parts but unfortunately, due to the tragic sudden death of Wilson, this work was

[2]n.d. stands for not dated document

never completed).

Add.Ms.b.105 - 107A. Copy of Ramanujan's "Notebook2", Chapters XII - XXI, with G.N.Watson's proofs of Ramanujan's formulae [incomplete], n.d.

These are G.N.Watson's transcripts of Chapters XVI - XXI of the second of Ramanujan's Notebooks together with proofs of Ramanujan's formulae and gaps for proofs to be filled in later (as noted by Prof.R.A.Rankin, in 1969). In Add.Ms.b.105 Chapters XII - XV are covered. In Add.Ms.b.106 Chapters XVI - XVIII and in Add.Ms.b.107 Chapters XIX and XX are covered by G.N.Watson.

Add.Ms.b.107A. Copy of Ramanujan's three Quarterly Reports to the Board of Studies in Mathematics, 5 August and 7 November 1913 and 9 March 1914.

Add.Ms.b.107A contains 282 sheets which are B.M.Wilson's Mss notes on Ramanujan's Notebooks.

Add.Ms.b.107B. Investigations on Chapters II - VII of Ramanujan's "Notebook 2" by B.M.Wilson. n.d.

Add.Ms.b.107C. Account of S.Ramanujan and his work by B.M.Wilson, n.d. List of corrections to Chapters I - X of Ramanujan's " Notebook 2", with references to published work of other mathematicians, by B.M.Wilson, n.d. Notes and proofs of results in Chapters II - XIII of Ramanujan's "Notebook 2" by B.M.Wilson, n.d. General account of Ramanujan's life and work by B.M.Wilson, n.d.

Appendix 3. Personal file of S. Ramanujan at the National Archives and papers at the Tamil Nadu Archives.

This is the 'Personal File of S. Ramanujan, F.R.S. (1888 - 1920)', handed-over by the Madras Port Trust authorities to the National Archives of the Government of India at New Delhi in the 1950s. It is a fullscape size volume with 206 pages (133A, 141A, 159A, 23 and 167 are blank sheets). The original documents are deacidified and laminated (i.e. pasted in between translucent tracing paper) by Mr. Mast Ram Sharma and dated 30 November 1964 and checked on 22 October 1965. The contents of the volume are not catalogued. The pages are numbered in pencil and the contents are as noted below:

1. p1. Letter from S. Ramanujan to the Chief Accountant of Port Trust Madras, dated 9th Feb. [?] (Application for a Clerkship)

2. p3. Letter of recommendation from E.W. Middlemast, Acting Principal, dated 21-9-1911

3. p.5. Unsigned sanction letter (Class III, grade IV in accounts section), dated 9-2-12

4. p.7. Appointment order to S. Ramanujan [office copy of the Chief Accountant] (offering Rs.25/- per month from 1-3-12)

5. pp.9-11. Letter from C.L.T. Griffith to Sir Francis Spring, 12-11-12

6. pp.13-15. Letter from the Director of Public Instruction to Sir Francis Spring, 14-11-12

7. pp.17-20. Letter from the Accountant General to C.L.T. Griffith, 27-11-12

8. pp.21-22. Letter from C.L.T. Griffith to Sir Francis Spring, 28-11-12

9. pp.23-24. Faded and unreadable

10. p.25. Memo of the Madras Port Trust to Mr.S. Narayana Aiyar, 1-12-[?]

11. p.26. Faded and unreadable

12. pp.27-29. Letter from M.J.M. Hill. [It is the letter which contains the following : "One thing however is clear. Mr.Ramanujan has fallen into the pitfalls of the very difficult subject of Divergent series." And :

$$\text{“}1+2+3+\cdots+\infty = -\frac{1}{12}$$

$$1^2+2^2+3^2+\cdots+\infty^2 = 0$$

$$1^3+2^3+3^3+\cdots+\infty^3 = \frac{1}{240}$$

All these have infinity for their sum." These sums can be understood today in the context of analytic continuation of the Riemann ζ function $\zeta(s) = \sum_{n=0}^{\infty} \frac{1}{n^s}$, for negative values of s. These results find use in the field of String Theory]

13. pp.30-31. Letter from M.J.M. Hill to Prof.C.L.T. Griffith, 3-12-1912

14. pp.32-34. An unsigned letter from the University of London to Mr.C.L.T. Griffith [from M.J.M. Hill ?], dated 7-12-1912

15. pp.35-37. Letter from C.L.T. Griffith to Sir Francis Spring, 5-1-13

16. pp.39-52. First letter from G.H. Hardy to S. Ramanujan, 8 Feb. 1913 [pp. 40, 42, 44, 46, 48, 50 and 52 are blank]

17. pp.53 and 55. Further notes suggested by Mr.J.E. Littlewood [8.2.13 ?]

18. p.57. Typewritten “chains of acting arrangements”, 11-2-13

19. p.59. Letter from Mr.Hanumantha Rau to Mr.Narayana Aiyar, 13-3-1913 [calling a meeting of the Board of Studies in Mathematics to discuss “What we can do for S. Ramanujan”]

20. p.61. Regarding Dr.Gilbert Walker's visit, [31-3-17, ?]

21. Typewritten from S. Ramanujan to the Registrar of the University of Madras, 12-4-1913 [accepting the Rs.75/- per month scholarship]

22. Letter of Chief Accounts Commissioner (CAC), 11-4-13 [recommending 2 years leave on loss of pay for S./Ramanujan from 1st May 1913]

23. p.67-68. Slip Acknowledging letter about scholarship to S. Ramanujan – conveying the same to Mr.Mallet of University Office – Arthur Davies' post card, 7-6-13

24. p.69. Mr.S. Narayana Aiyar's note, 31-X-13

25. pp.71-73. CAC S. Narayana Aiyar's note, 12-3-[1913 ?]

26. pp.75-78. 4 typewritten (notebook size) pages from Mr.Narayana Aiyer [?] to Mr.Cotterell, 5 Feb.[1914 ?]

27. p.79. Letter from the Private Secretary of His Excellency [?] to Sir Francis Spring, 5 Feb. 1914

28. p.81. Extract [typewritten] from the 'Madras Mail, 13 May 1914

29. 'Notes' of S. Narayana Aiyar, CAC, Madras Port Trust, 15-4-15

30. Letter to the Registrar from Sir Francis Spring [?, hand written, on plain paper]

31. p.87. Notes of S. Narayana Aiyar, 18-12-15

32. p.89. Note of S. Narayana Aiyar [to Sir Francis Spring ?], 15-XII-191?

33. p.91. Typed copy of the above, 15-XII-191?

34. p.93. Letter [ref.no.7923] from Francis Dewsbury to Sir Francis Spring, 16-12-1915

35. p.95. Ref.No.7923 forwarded to the University of Madras by S. Narayana Aiyar, 20-12-15

36. p.97. Extract from a letter from S. Ramanujan dated 11 Nov. 1915

37. p.99. Letter from Francis Dewsbury to Sir Francis spring, 17 Jan. 1916

38. p.101. Note of S. Narayana Aiyar suggesting extension of leave on loss of pay, 18-X-16

39. p.103. Record of a telephone message asking for Ramanujan's date of birth by the Registrar of the University of Madras, 14-3-17 and S. Narayana Aiyar's reply note dated 14-5-17

40. p.105. Letter from the Registrar of the University of Madras, regarding Ramanujan's father (whether he was alive and whether he was a District Munisiff), 14-5-17

41. pp.109-112. Letter from S. Ramanujan [to Narayana Aiyar ?], 11-11-1915

42. p.113/1 and 113/2. Letter from S. Ramanujan [to Narayana Aiyar ?], 11-11-1915

43. p.115. letter from Nellore, 6 Dec.1915

44. p.119. Newspaper cutting from "Thursday Evening", March 7 [?]

45. p.121. Note from Sir Francis Spring and reply of S. Narayana Aiyar, 12-3-18

46. p.123. Letter from P.V. Seshu Aiyar to S. Narayana Aiyar, 17-3-18

47. p.125. S. Narayana Aiyar's Notes reg. the FRS felicitation function at Presidency College during the week, 18-3-18

48. p.127 Newspaper cutting from The Statesman, 10-12-1918

49. p.129. Newspaper cutting from the Madras Times, March 30,1919

50. p.131. Newspaper cutting from The Madras Mail, with a note of CAC, March 29, 1919

51. pp.133-155. Typewritten letter (sd.)Sir Francis Spring, 20 Dec.1915

52. p.157 Note of Mr.S. Narayana Aiyar, CAC, n.d.

53. p.158. Article entitled : "A Famous Madras Mathematician", in the newspaper The Madras Times, April 6, 1919

54. p.161. Letter to S. Ramanujan in Kumbakonam from the Chief Accountant, madras Port Trust, 21-X-19

55. p.163. Note granting leave without pay for 5 years to S. Ramanujan and terminating his services on 30-4-1918, dated 21-X-19

56. p.165. Letter to the Chief Accountant, Madras Port Trust, reg. Provident Fund + subscription (amounting to Rs.103-annas 12-paise 8) from S. Ramanujan, 12-1-20

57. p.167. Faded and unreadable

58. From the Chief Accountant, Madras Port Trust, to S. Ramanujan at Chetput, 20-1-[20 ?]

59. From the Chief Accountant, Madras Port Trust to S. Ramanujan at Chetput, n.d.

60. p.173. Newspaper cutting from The Madras Mail of Monday evening announcing the death of S. Ramanujan, April 26,1920

61. PP.175-176. Letter from Sir Francis Spring in London to S. Narayana Aiyar, 15-2-21

62. Typewritten letter from S. Narayana Aiyar to Sir Francis Spring, 10-3-21

63. p.179. Letter from Ramachandra Raw [asking for details for article], n.d.

64. p.181. Newspaper cutting from The Hindu dated dec.11, 1947, announcing the "Death of Hardy"

65. p.183. Newspaper cutting of the article on "Prof. Hardy" by J.B.S. Haldane, Jan. 25, 1945

66. Newspaper cuttings from the Hindu dated April 18, 1948 – 'Tribute to Genius of Ramanujan" and "Portrait Unveiled"

67. p.187. Newspaper cuttings from The Mail dated June 13, 1948 – "Late Mr.Srinivasa Ramanujan" and "Portrait unveiled at Port trust Office"

68. p.189. Letter of S. Narayana Aiyar, 18-6-48

69. p.191. Article : "Srinivasa ramanujan : India's Great Mathematician" by S. Krishnan in " Wealth & Welfare", Madras, dated Aug.1, 1947

70. p.195. Typewritten article on "Mr.S. Ramanujan, F.R.S.", by S. Narayana Aiyar, CAC

71. p.197-201. Typewritten address on the occasion of the unveiling of the portrait of S. Ramanujan by the Chief Accountant [of Madras Port trust ?]

72. p.203. Newspaper cutting from The Hindu dated Nov.22, 1948 – "Tribute to late S. Ramanujan"

73. p.205. "American Tribute to late Ramanujan", Newspaper cutting dated Dec.24, 1949 – Dr.Marshal Stone's interview with Globe.

Files in Tamil Nadu Archives

The following are the Madras Government (Educational) Files in the Tamil Nadu Archives, Chennai - 600 008, pertaining to Srinivasa Ramanujan:

S.No	G.O.No.	Date of G.O.	Regarding	Page
1	291	7-4-1913	F. Dewsbury to the Secretary	
2	182	12-2-1914	Scholarship sanction	
3	289	18-3-1914	C. W. E. Cotton to the Chief Secretary	
4	1308	20-11-1914	– do –	
5	1441	20-12-1914	F. Dewsbury to the Secretary	
6	345	13-3-1917	F. Dewsbury to the Secretary	
7	475	11-4-1918	F. Dewsbury to the Secretary	
8	408	2-4-1919	F. Dewsbury to the Secretary	
9	1072	5-9-1919	Komalatammal to the Secretary	
10	1072-73	17-9-1920	Pension matters to the family	5
11	132	26-1-1921	Pension matters	2
12	487	13-3-1921	Registrar to the Secretary	
13	1160	11-8-1921	E. S. Montagu's grant of allowance	

References

The principal references are denoted by Roman numerals:

I. "Ramanujan: Letters and Reminiscences", Memorial Number, Vol.I. ed. P.K. Srinivasan, Muthialpet High School, Madras. 1968.

II. "Ramanujan : An inspiration"; Memorial Number, Vol.II, ed. P.K. Srinivasan, Muthialpet High School, Madras, 1968.

III. "Collected Papers by Srinivasa Ramanujan", edited by G.H. Hardy, P.V. Seshu Aiyar and B.M. Wilson Chelsea, New York, 1962; first published by Cambridge University Press, Cambridge.

IV. "Ramanujan: Twelve Lectures on subjects suggested by his life and work", G.H. Hardy, Chelsea, New York, 1940.

V. "Professor Srinivasa Ramanujan Commemoration Volume", ed. Ramananda Bharathi, published by the Professor Srinivasa Ramanujan International Memorial Committee", Madras, 1974.

VI. "Ramanujan: The Man and the Mathematician", S.R. Ranganathan, Asia Publishing House, 1967.

VII. "Notebooks of Srinivasa Ramanujan", (facsimile edition) 2 Volumes, Tata Institute of Fundamental Research, Bombay, 1957; Narosa, New Delhi, 1987.

VIII. "Ramanujan Notebooks", Parts I (1985), II (1989), III (1991), IV (1994) and V (1997), Bruce C. Berndt, Springer-Verlag, New York.

IX. "Srinivasa Ramanujan, 1887 – 1920: A tribute", ed. by K.R. Nagarajan and T. Soundararajan, Macmillan India Ltd. (1988) and articles therein of K. Srinivasa Rao, G.E. Andrews, R. Askey, B.C. Berndt, R.P. Agarwal, A. Verma, S. Bhargava, M.V. Subba Rao and T. Soundararajan.

X. "The Man Who Knew Infinity: A Life of the Genius Ramanujan", Robert Kanigel, Charles Scribner's Sons, New York (1991); Indian edition published by Rupa & Co. (1994).

XI. "Ramanujan: Letters and Commentary", Bruce C. Berndt and Robert A. Rankin, American Mathematical Society and London Mathematical Society (1995); also Indian Edition with a Preface, Additions to the Indian Edition and Errata, by K. Srinivasa Rao, published by Affiliated East West Press Pvt. Ltd. (1997).

XII. "A Mathematician's Apology", G.H. Hardy (with a Foreword by C.P. Snow), Cambridge University Press (1976), first published in 1967.

XIII. "Collected Papers of G.H. Hardy", including joint papers with J.E. Littlewood and others, ed. by a Committee appointed by the London Mathematical Society, Volumes I (1966), II (1967), III (1969), IV (1969), V (1972), VI (1974) and VII (1979).

XIV. "Development of Elliptic Functions according to Ramanujan", K. Venkatachaliengar, Madurai Kamaraj University, Technical Report 2 (1987).

XV. "Ramanujan Papers", K. Srinivasa Rao, Math. Student (1997).

XVI. "Ramanujan Revisited", ed. by George E. Andrews, Richard A. Askey, Bruce C. Berndt, K.G. Ramanathan and Robert A. Rankin, Acad. Press, Inc., (1988).]

* * * * * * * *

1. C.V. Rajagopalachari, in I. p83-84. Also, in [98], p.34.
2. K.S. Viswanatha Sastri, in I, p.89-93.
3. N. Govindarajan, in VI. p62-63, also in I, p.104-105.
4. See I, p.94, 95, 120, 121.
5. N. Hari Rao, in I, p.120-123.
6. V. Ramaswamy Iyer, in I, p.129.
7. R. Ramachandra Rao, in I, p.126-127.

8. Griffith to Sir Francis Spring, in I, p.50.

9. M.J.M. Hill to C.L.T. Griffith, in I, p.53.

10. C.P. Show in his Foreword to G.H. Hardy's: "A Mathematician's Apology", Cambridge University Press (1976) p.30 [XII].

11. In the words of C.P. Show: "Hardy was not the first eminent mathematician to be sent the Ramanujan manuscripts. There had been two before him, both English, both of the highest professional standard. They had each returned the manuscripts without comment. I don't think history relates what they said, if anything, when Ramanujan become famous". As for their identity, Snow adds that: "out of chivalry Hardy concealed this in all that he said or wrote about Ramanujan". (p.33 of ref.10). However, the names are given by A. Nandy (in "Alternative Sciences". Allied Publishers. New Delhi. 1980) who claims the two to be H.F. Baker and E.W. Hobson. (also see VIII, Part I, p.3).

12. Gilbert T. Walker to Francis Dewsbury, in I, p.55.

13. p.xxiii of III, and also reproduced in XI.

14. "Ramanujan's first letter in Hardy's hands". C.P. Snow, in I, p.157-158.

15. E.H. Neville, in I, p.138-141.

16. E.H. Neville to Dewsbury, in I, p.59-60.

17. Littlehailes to Dewsbury, in I, p.61-63.

18. Sir Francis Spring to C.B. Cotterell, in I, p.64-65.

19. Letter 2 to R. Krishna Rao, in I. p.4-7.

20. "Srinivasa Ramanujan". J.R. Newman, in 'Mathematics in the modern World'. W.H. Freeman & Co. (1968) 73-76.

21. p.xxx in III.

22. Letter 4 to R. Krishna Rao in I, p.12-19.

23. Letter 1 to S.M. Subramainan in I, p.20-17.

24. P.C. Mahalanobis in I, p.145-148.

25. Hardy to Dewsbury in I, p.76-77.

26. Footnote on p.xxxiii in III.

27. p.53, vol. II of [VII].

28. Hardy to Subramanian in I, p.68-75.

29. Janaki Ramanujan in I, p.159-161 (Tamil), p.171-172 (English translation).

30. T.K. Rajagopalan in I, p.167 and in VI. p.87, R. Srinivasan in I, p.165-166: R. Radhakrishnan Ayyar in VI, p.73.

31. Copy of Ramanujan's letter to the Registrar, University of Madras, in the list of plates, ref. VI. Also reprinted on p.xix of III.

32. K. Ananda Rao in I, p.143-144.

33. Letter to Narayana Iyer, in I, p.32-33.

34. p.7 of VIII, Part I.

35. Footnote on p.xxv in III.

36. p.9. of IV.

37. "A chapter form Ramanujan's notebook". G.H. Hardy, Proc. Cambridge Phil. Soc. **21** (1923) 492-503.

38. p.57 of VI.

39. G.N. Watson, "Ramanujan notebooks", J. London Math. Soc. **6** (1931) 137-153.

40. p.5 of VIII, Part I.

40a. p.139, Entry 4, of Vol. 2 of VII; pp. 10-12 of II.

41. p. 87 – 90 of IV.

42. H. Gupta, "A table of partitions", Proc. London Math. Soc. (2) **39** (1935) 142-149; ibid **42** (1937) 546-549; H. Gupta, "On a conjecture of Ramanujan", Proc. Indian Acad. Sci. (A) **4** (1936) 625-629.

43. S. Chowla, "Congruence properties of partitions", J. London Math. Soc. **9** (1934) 247.

44. W. Krečmar, "Sur les propriétés de la divisibilité d'une fonction additive", Bull. Acad. Sci. URSS **7** (1933) 763-800.

45. p.xxxv in III.

46. S Ramanujan: "Some properties of $p(n)$, the number of partitions of n", Proc. Cambridge Phil. Soc. **19** (1919) 207-210.

47. "Ramanujan's Congruences". in IV, p.87-90.

48. p.91 in IV. Refer articles of Arun Verma, p.66 and S. Bhargava, p.75, in IX.

49. C. Adiga, B.C. Berndt, S. Bhargava and G.N. Watson. "Chapter 16 of Ramanujan's second notebook: Theta-functions and q-series", Memoir No. 315, American Math. Soc., Providence 1985.

50. p.15 of IV.

51. J.E. Littlewood, p.154-156 in I. Also G.E. Andrews, "The Theory of Partitions, Encyclopaedia of Mathematics and its Applications, Vol. **2**, Addison-Wesley, Reading, Massachusetts, 1976, p.68-69.

52. For a brief discussion on the Hardy-Littlewood-Ramanujan 'circle method', see R. Bellman: "A Brief Introduction of Theta Functions", Holt, Rinehart and Winston, New York, 1961, p.14.

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Notes

Biographical: Often Ramanujan is compared with Euler, Gauss or Jacobi for *natural genius*. In this section, brief biographical notes are provided on these mathematicians:

Euler, Leonhard (1707 – 1783): Switzerland's foremost scientist and considered as one of the three greatest mathematicians of modern times, along with Gauss and Riemann. His important contributions are to practically every area of pure and applied mathematics. He was the author of more than 200 articles and three treatises on analysis. Though he became totally blind, in 1771, he continued to contribute to optics, algebra and the theory of moon's motion. He made the concept of a function fundamental in analysis. He established the modern form and notation for the trigonometric and logarithmic functions. The equations $e^{\pm ix} = \cos x \pm i \sin x$ are called as Euler's equations. Through his work, the symbols: e, π and i became common for all mathematicians and they are linked through the fundamental relation: $e^{i\pi} + 1 = 0$. The notations for trigonometric functions $\sin x, \cos x$ and the use of $f(x)$ for a general function and the symbol Σ for summation are due to Euler. The constant

$$\gamma = \lim_{m\to\infty} \{1 + \frac{1}{2} + \frac{1}{3} + \ldots \frac{1}{m} - \log m\} = 0.5772157\ldots$$

is known as *Euler's constant* (or, as Mascheroni's constant).

Gauss, Karl Friedrich (1777 – 1855): German mathematician, acknowledged to be one of the three leading mathematicians of all time, the others being Archimedes and Sir Issac Newton. His outstanding work includes the discovery of the method of least squares, the discovery of the non-Eucledian geometry and important contributions to the theory of numbers. He surprised his teacher at the age of ten by summing 1 to 100 mentally with the help of the formula: $\sum_{i=1}^{n} i = n(n+1)/2$, when the teacher expected him to be occupied with the task for a long time, just as others were expected to do it the hard way! In 1796, he discovered how to construct a regular polygon of 17 sides using only a compass and a straight edge. In fact, he proved that a regular polygon with n sides is constructible if and only if n is a distinct prime number of the form $p_k = 2^{2^k} + 1$. Thus, when $k = 0, 1, 2, 3$, the corresponding numbers $p_k = 3, 5, 17, 257$ are prime umbers and so regular polygons with these number of sides are constructible. This result happens to be the first item in a mathematical diary he kept until 1814. Gauss's diary, like the notebooks of Ramanujan, makes it possible to verify the priority of Gauss in many of the discoveries he did not publish. This little booklet of only 19 pages, considered as one of the most precious documents in the history of science, was unnoticed till 1898, and though not lost, was found among family papers with one of Gauss's grandsons. It contains 146 concise statements of the results of his researches, between 1796 to 1814. Among the unpublished discoveries of his are: non-Eucledian geometry and non-commutative algebras.

His doctoral thesis contained the first proof of the fundamental theorem of algebra,

which states that every algebraic equation has a root of the form $a + i\ b$ where a and b are real numbers and i is $\sqrt{-1}$. In general, numbers of the form $a + i\ b$ are called as *complex numbers*, and, in particular, when a and b are integers, the numbers $a + i\ b$ are called as *Gaussian numbers*. Complex numbers are represented in a plane called the Gaussian plane.

Gauss published, in 1801, *Disquisitiones Arithmeticae*, his greatest treatise, in which Gauss created the modern rigourous approach to mathematics. It contains the new algebra of congruence. He introduced the $\equiv$ symbol for congruence and the notation $a \equiv b\ (mod\ m)$, which has since become a standard in the theory of numbers. The *law of quadratic reciprocity* discovered by him was called *the gem of arithmetic* by him: *if p and q are prime numbers, then either $x^2 \equiv q\ (mod\ p)$ and $x^2 \equiv p\ (mod\ q)$ are both solvable or both unsolvable, unless both p and q are of the form $4n + 3$, in which case one is solvable and the other is not.* Gauss's *Arithmetical Investigations* is divided into seven parts: Congruences in general, Congruences of the first degree, Residues of powers, Congruences of the second degree, Quadratic forms, Applications and Divisions of the circle. This work (judged extremely hard to read) has been disseminated by his successor at Göttingen, Peter Gustav Dirichlet (1805 – 59).

In 1811, Gauss extended the calculus to functions of a complex variable and discovered a fundamental theorem of integration: if a function $f(z)$ is analytic at all points on and within a closed curve C in the Gaussian plane, then the integral of $f(z)$ along C is zero. Since he did not publish this theorem, it is known today, after its rediscoverer, as Cauchy's integral theorem. In Jan. 1812, recognizing the need for convergence of infinite series, he published his comprehensive, important work on hypergeometric series (see ch.IV, pp. 110 – 119 of this book.). Gauss wrote extensively on the theory of errors of measurement in his *Theoria Motus* and subsequent works that the normal frequency distribution which is the familiar bell-shaped curve is often referred to as the Gaussian distribution (and finds a place on the face of the 10 DM currency note of Germany, see p. 175). Gauss is considered as the effective founder of differential geometry. He introduced the concept of the Gaussian or total curvature of a surface, as a measure of curvature (which remains unchanged under a continuous deformation of a flexible and inextensible surface).

Gauss developed the theory of elliptic functions, though he did not publish his results. The remarkable property of double periodicity of these functions was rediscovered by Niels Abel and Karl Jacobi (1827 – 29).

A unit of magnetism is named after Gauss. The eyepiece used for autocollimation in spectrometers and refractometers is designed by him. He invented the heliotrope, the biflar magnetometer and with Weber devised the mirror galvanometer. In 1834-35, Gauss and Weber invented an apparatus for transmission of messages. This telegraph system of Gauss and Weber was destroyed by lightning in 1845. There exists a Gauss-Weber, life-size, bronze monument in Göttingen where the period when Gauss was the Astronomer Royal marked a golden era of science in that country. (Further reading: *Carl Friedrich Gauss: A Biography* by Tord Hall, The MIT Press, Cambridge, Massachusetts (1970).)

Jacobi, Karl Gustov Jacob (1804 – 51): A German mathematician who made important contributions to many branches of mathematics. With Niels Abel, Jacobi was

the discoverer of elliptic functions, in 1829, which led him to further generalizations of the concept for Abelian functions, in 1832 – 35. Jacobi transformed William Hamilton's theory of mechanics into what is known today as the Hamilton-Jacobi theory, which is considered as the most elegant formulation of classical mechanics. Jacobi's work made determinants generally acceptable to all mathematicians. Jacobi visited Gauss on several occasions to discuss his recent researches with him and *'each time Gauss pulled 30-year-old manuscripts out of his desk and showed Jacobi what Jacobi had just shown him'*! It is therefore not surprising that Jacobi is thought to never have come in close contact with Gauss.

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algebraic number: A *complex number a* is said to be an algebraic number if it satisfies a polynomial equation over the field of rational numbers. For example, $\sqrt{2}$ is an algebraic number as it satisfies the polynomial equation $x^2 - 2 = 0$.

American Mathematical Society: The AMS was founded in 1888. About 30000 individuals, colleges, universities and other institutions are among its members. (The author of this book was a member of the AMS during 1993 - 96.) This society publishes the *Proceedings*, the *Memoirs* and the *Transactions* of the AMS, as well as the extremely useful *Mathematical Reviews*, a reviewing and abstracting journal; the two monograph series: *Colloquium Publications* and *Mathematical Surveys*; and the *Bulletin* of the AMS. Nine sectional meetings, a summer meeting in September and an annual meeting in December are held by the AMS. Early each summer, the society holds a symposium in applied mathematics. The AMS and LMS combined publication: *Ramanujan: Letters and Commentary* [XI] by Bruce C. Berndt and Robert A. Rankin (1995) has been adjudged as "Outstanding Academic Book of 1996" (Choice). An Indian edition published by Affiliated East West Press Pvt. Ltd., with a *Preface to the Indian edition* and *Additions to the Indian edition* was brought out by the author of this book. (Details of the AMS may be obtained from the Society's Web site: http://www.ams.org)

analytic function: A function of the complex variable $f(z)$ is said to be *differentiable* at $z = a$ if the limit,

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

exits independent of the path of $h \to 0$ in the complex plane. The limit is denoted by $f'(a)$ and is called the derivative of $f(z)$ at a. A function $f(z)$ is said to be analytic, in a region of the complex plane, if it is differentiable in that region. A function $f(z)$ with the property that a power series expansion of the form

$$f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n$$

is valid in some neighbourhood of the point z_0, where $a_n = f^{(n)}(z_0)/n!$, $f^{(n)}(z_0)$ being the nth derivative of the function at z_0, is said to be analytic at z_0. The series is called the Taylor series of $f(z)$ at z_0.

analytic number theory: It is a branch of number theory. This subject deals with the problems of distribution of primes, the behaviour of number-theoretic functions, and the theory of algebraic and transcendental numbers.

arithmetical function: A real or complex valued function defined on the set of positive integers is called an arithmetical function.

Association of Mathematics Teachers of India (AMTI): This Association was registered under the Society's Act on Oct. 29, 1965. Its first President was the distinguished Dr. A. Narasinga Rao and he was followed by Prof. P.L. Bhatnagar. The present President is Prof. R.C. Gupta and the present Executive Chairman is Prof. G. Rangan (of the Ramanujan Institute of the University of Madras). A large number of mathematics teachers from all over India are members of AMTI and take part in interactive annual meetings of the association. The AMTI conducts the mathematics Olympiads in India and helps the talented students in every possible way. The student K. Soundararajan (ref. p.177 of this book) is one of those who has been nurtured and helped by the AMTI. The *Mathematics Teacher* is its official journal.

astrophysics: The study of stellar evolution, stellar dynamics and inter-stellar material; or, the physics of astronomical bodies like: planets, stars, galaxies, etc.

asymptotic formula: An identity of the form $f(x) \sim g(x)$ which means that $f(x)$ is equal to $g(x)$ as $x \to \infty$. We say that $f(x)$ is asymptotically equal to $g(x)$. For example, $x^3 + x + 1 \sim x^3$.

Bernoulli numbers: The contemporary definition for Bernoulli numbers, B_n, is

$$\frac{x}{e^x - 1} = \sum_{n=0}^{\infty} \frac{B_n}{n!} x^n, \qquad |x| < 2\pi.$$

Beta function: A function of two variables p and q which, for $p, q > 0$, is defined by

$$B(p,q) = \int_0^1 x^{p-1}(1-x)^{q-1}\,dx = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}.$$

Black hole: Black holes are regions of space-time where the gravitational field is so strong that even light cannot escape them. Inevitably, black holes form as a result of the gravitational (matter) collapse of a star, whose mass is sufficiently large (of the order of a few solar masses), once it has burnt its nuclear fuel. They can also be formed by the gravitational collapse of the inner regions of a galaxy, or by collisions.

Carr's Synopsis: A copy of the Carr Synopsis is available at the Library of the University of Madras. (It is reported that the copy at the Town High School in Kumbakonam, used by Ramanujan, has been lost about 5 years ago. Private communication from a Japanese Professor who visited Kumbakonam, in Feb. 98).

Cauchy's theorem: Let f be analytic in a region Ω of the complex plane. Let γ be a closed curve in Ω. Then for all points a in Ω which do not lie on γ we have:

$$f(a) = \frac{1}{2\pi i} \int_\gamma \frac{f(z)}{z-a} dz.$$

circle method: This method was developed by Ramanujan and Hardy when they were working on the partition problem. The problem is to find an asymptotic formula for $p(n)$, the number of partitions of n. Euler proved that

$$\prod_{j=1}^{\infty}(1-z^j)^{-1} = \sum_{n=1}^{\infty} p(n)z^n.$$

Let $f(z)$ denote the infinite product on the left side. The singularities of $f(z)$ are the roots of unity and lie densely on the unit circle $|z| = 1$. Thus $f(z)$ has the unit circle as its circle of convergence. By Cauchy's residue theorem, we have

$$p(n) = \frac{1}{2\pi i}\int_{|z|=r} \frac{f(z)}{z^{n+1}}dz$$

for $0 < r < 1$. Thus the problem of estimating $p(n)$ is equivalent to estimating an integral. This beautiful idea has turned out to be applicable to a large class of related problems – for instance, Waring's problem in Additive Number Theory. (cf: Some unsolved problems in Number Theory, K. Ramachandra, Resonance, May 1997, Vol. 2, No. 5, p. 78).

combinatorial analysis: It is a branch of mathematics which deals with the problems of choosing and arranging the elements of certain (normally finite) sets according to some prescribed rules.

complex numbers: a number of the form $x + iy$, where x and y are arbitrary real numbers, is called a complex number. Gauss is accredited for suggesting this nomenclature. When $x = 0$, the number iy is called a pure imaginary number. The complex number $x+iy$ is a number pair (x, y) for which elementary algebraic rules of addition, subtraction, multiplication and division are defined as:

$$(x, \pm y) = x \pm iy$$

$$(x_1, y_1) \pm (x_2, y_2) = (x_1 \pm x_2, y_1 \pm y_2)$$

$$(x_1, y_1) \cdot (x_2, y_2) = (x_1x_2 - y_1y_2, x_1y_2 + y_1x_2)$$

$$\frac{(x_1, y_1)}{(x_2, y_2)} = \left(\frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2}, \frac{x_1y_2 - y_1x_2}{x_2^2 + y_2^2}\right)$$

If $z = x+iy$ is a complex number then x and y are called the real and the imaginary parts of z and we write $x = Re(z)$ and $y = Im(z)$.

composite number: A *natural number* which is not a *prime number* is called a composite number. The number 1 is neither a prime nor composite. Every composite number can be written in only one way as the product of prime numbers, which are called its prime factors. For example: $6 = 2 \times 3$, $9 = 3 \times 3$.

congruence: If a number m divides the difference of two numbers a and b, viz. $(a - b)$ or $(b - a)$, exactly then we say that a is congruent to b modulo m and we write

$$a \equiv b(mod\ m)$$

This relation is called a *congruence* and m is called the modulus of the congruence. The number b is called the residue of a modulo m. The congruence $a \equiv 0(mod\ m)$ means that a is divisible by m, for example: $35 \equiv 0(mod\ 5)$ means 35 is divisible by 5 and 0 is the residue of 35 modulo 5.

continued fraction: Any expression of the form:

$$C(x) = \frac{1}{1} + \frac{x}{1} + \frac{x^2}{1} + \frac{x^3}{1} + \cdots \equiv \cfrac{1}{1+\cfrac{x}{1+\cfrac{x^2}{1+\cfrac{x^3}{1+\cdots}}}}$$

for $|x| < 1$, is called a continued fraction. (see p.41 and p.62 of this book).

convergent sequence: The sequence (a_n) is said to converge to the limit a if for every $\epsilon > 0$, there is a positive integer N such that $n \geq N$ implies $|a_n - a| < \epsilon$. We write $\lim_{n\to\infty} a_n = a$. For example: the sequence $(\frac{1}{n})$ is convergent and converges to 0. A sequence which does not converge is said to *diverge*. For example: the sequence (n) is divergent.

cosmic ray: Earth is constantly bombarded by extra terrestrial radiation consisting predominantly of protons and subnuclear particles of high energies. Early workers gave this phenomenon the name cosmic rays. Many important discoveries, including the production and identification of several new 'elementary' particles called mesons and hyperons, were made from cosmic rays, especially before the advent of man-made high energy particle accelerators.

Diophantine equations: Certain equations in number theory are called Diophantine equations after Diophantus who lived in Alexandria around 250 A.D. It is said that he published a series of books, *Arithmetica*, in 13 volumes, which were lost for more than a thousand years and that they were found again around 1570. Apparently, the early volumes introduced, for the first time, negative numbers, algebraic notation and equations, while the later volumes dealt with number theory. His books are in the style of problems and solutions.

Dirac's theory of electrons: Prof. P.A.M. Dirac, considered one of the great theoretical physicists of this century, applied the techniques of relativistic mechanics to quantum theory and arrived at the relativistic wave equation for the electron, since named after him as the Dirac equation. The solution of this equation predicted negative as well as positive energy states. Dirac interpreted that, in the absence of an external electric field, the negative energy states are always filled. When an appropriate field is applied, Dirac predicted that the transition of a particle from a filled negative energy state into an unfilled positive energy state takes place. This leads to pair production and the resulting positively charged particle (in all other aspects like an electron) is called the positron, the first anti-particle predicted by Dirac from his theory of the electron.

Dirichlet series: A series of the form

$$\sum_{n=1}^{\infty} \frac{f(n)}{n^s},$$

where $f(n)$ is an arithmetical function, is called a Dirichlet series with coefficient $f(n)$.

doubly periodic function: A function of a complex variable $f(z)$ is called a doubly periodic function of z if $f(z) = f(z + 2\omega_1) = f(z + 2\omega_2)$, where ω_1, ω_2 are complex numbers for which the ratio $\frac{\omega_1}{\omega_2}$is not a real number. $2\omega_1, 2\omega_2$ are called the periods of the function.

Euler-Maclaurin formula: The Euler-Maclaurin formula was indeed a favourite tool of Ramanujan. If the function f has $2n + 1$ continuous derivatives on $[\alpha, \beta]$, where α and β are integers, then

$$\sum_{k=\alpha}^{\beta} f(k) = \int_{\alpha}^{\beta} f(t)dt + \frac{1}{2}\{f(\alpha) + f(\beta)\} +$$

$$+ \sum_{k=a}^{n} \frac{B_{2k}}{2k!}\{f^{(2k-1)}(\beta) - f^{(2k-1)}(\alpha)\} + R_n,$$

where, for $n \geq 0$,

$$R_n = \frac{1}{(2n+1)!} \int_{\alpha}^{\beta} B_{2n+1}(t - [t]) f^{(2n+1)}(t)dt,$$

where $B_n(x), 0 \leq n < \infty$, denotes the n^{th} Bernoulli polynomial.

false theta function: They are the *theta functions* with irregular/wrong signs.

F.R.S.: Fellow of the Royal Society, London. The Royal Society was started in the year 1645. Ramanujan was the second Indian but the first Indian mathematician to have been honoured with the fellowship. Amongst the Indian Fellows of the Royal Society are : Prof. C.R. Rao, Statistician, Emeritus Professor, Pennsylvania State University; Prof. C.S. Seshadri, SPIC Mathematical Institute, Madras; Prof. M.S. Narasimhan, ICTP, Trieste, Italy.

Fields Medal: This is awarded in the name of John Charles Fields who was a Professor of Mathematics, University of Toronto, Canada. His successful organization of the International Mathematical Congress in Toronto, in 1924, ended with a surplus of funds which provided the initial funding for the Fields Medals. These were later further endowed from his estate. In 1932, the International Mathematical Congress at Zürich adopted his proposal and the Fields medal awards were first made at the next congress held at Oslo in 1936. The medals have been traditionally awarded to mathematicians under 40 in compliance with Fields' wish that they be an encouragement for further work by the recipients. The Gold plated cast is 25cms. in diameter and the head appearing on the medals minted in the Royal Canadian Mint, is that of Archimedes. This award is the equivalent of the Nobel Prize, since there is no Nobel Prize for Mathematics! Unlike the Nobel Prizes which are awarded every year, the Fields Medals are awarded once in four years.

fundamental theorem of arithmetic: Every integer $n > 1$ can be represented as a product of prime factors in only one way, apart from the order of the factors.

Gamma function: It is defined as

$$\Gamma(z) = \int_0^{\infty} x^{z-1}e^{-x}dx, \text{ for } Re(z) > 0.$$

It is remarkable that Ramanujan has deduced the values of a large variety of known and new gamma function integrals.

Gauss hypergeometric series: A series of the form

$$F(\alpha,\beta;\gamma;z)=\sum_{n=0}^{\infty}\frac{(\alpha)_n(\beta)_n}{n!(\gamma)_n}z^n,$$

where $(\alpha)_n$ is called the Pochammer symbol. This series is not meaningful when γ is zero or a negative integer. The series is convergent for $|z|<1$, divergent for $|z|>1$ and for $|z|=1$, it is convergent if $Re(\gamma-\alpha-\beta)>0$ and divergent if $Re(\gamma-\alpha-\beta)<0$. If $\alpha=1$, and $\beta=\gamma$, then the series reduces to the geometric series: $1+z+z^2+z^3+\cdots$. Gauss who introduced this series, considered it as a function of four variables. The modern notation for the function on the left hand side of the above equation is ${}_2F_1(\alpha,\beta;\gamma;z)$ where the subscripts 2 and 1 represent the fact that the series has two numerator parameters and one denominator parameter. (ref. p.110 of this book).

generalized hypergeometric series: A series of the form

$${}_pF_q(\alpha_1,\cdots,\alpha_p;\beta_1,\cdots,\beta_q;z)=\sum_{n=0}^{\infty}\frac{1}{n!}\frac{(\alpha_1)_n\cdots(\alpha_p)_n}{(\beta_1)_n\cdots(\beta_q)_n}z^n,$$

where $(x)_n=x(x+1)\cdots(x+n-1)$ is the Pochammer symbol.

Goldbach's conjecture: Every integer $n>5$ is the sum of 3 primes. Euler showed that it is equivalent to the statement that: every even integer $2n>4$ is the sum of two primes.

highly composite number: A number whose number of divisors exceeds that of all its predecessors. Examples: $2,3,4,6,12,24,\ldots$ (ref. p.46 of this book).

imaginary numbers: While the equation $x^2-1=0$ has two roots, viz. 1 and -1, the equation $x^2+1=0$ has no roots in the domain of real numbers. Formally, the roots of the equation: $x^2=-1$ are $+\sqrt{-1}$ and $-\sqrt{-1}$. Euler, and others before him, denoted $\sqrt{-1}$ by the symbol i, an abbreviation for the word "imaginary" which was meant to indicate that they have no existence in the real world! Products of i follow the rules:

$$i.i=i^2=-1,\qquad i.i.i=i.i^2=-i,\qquad i.i.i.i=i^2.i^2=i^4=1.$$

I.M.S.: Indian Mathematical Society founded by Prof. V. Ramaswamy Iyer in the year 1907. A photograph of the first 9 presidents of IMS is reproduced on p.5 of this book.

infinite series: Let (a_n) be a *sequence* of real or complex numbers. The formal infinite sum $a_1+a_2+a_3+\ldots$ is called an infinite series or simply series. It is denoted by $\sum_{n=1}^{\infty}a_n$ or simply $\sum a_n$. With (a_n) we associate a finite sequence (s_n) where $s_n=a_1+a_2+\ldots+a_n=\sum_{i=1}^{n}a_i$. (s_n) is called the nth partial sum of $\sum a_n$. If (s_n) converges to s we say that the series converges and write $\sum a_n=s$. s is called the sum of the series. If (s_n) diverges the series is said to diverge. For example: $\sum\frac{1}{n^2}$ converges. The geometric series $\sum x^n$ converges to $\frac{1}{1-x}$ if $0\le x<1$ and diverges if $x\ge 1$. The series $1+1+1+\ldots$ and $\sum\frac{1}{n}$ are divergent.

infinity: Infinity is larger than the largest number one can choose. Thus it is not a number in the usual sense and it is denoted by the symbol ∞.

integers: $0,\pm1,\pm2,\pm3,\ldots$. We denote the *set* of integers by $\mathbf{Z}$.

integral: The simplest of all differential equations $\frac{dy}{dx} = f(x)$ has the solution: $y = \int f(x)dx + c$, where $\int f(x)dx$ is a symbol for the function with derivative $f(x)$. We can almost always give a meaning to the solution by writing it in the form:

$$y = \int_{x_o}^{x} f(t)dt + c,$$

where t is a dummy variable of integration. The crux of the matter is that this definite integral (an integral with specified limits of integration, as opposed to an indefinite integral in which the limits are unspecified) is a function of the upper limit x which always exists when the integrand is continuous over the range of integration and that its derivative is $f(x)$. (This statement is one form of the fundamental theorem of calculus).

interpolation: It is a process of estimating the values of a function $y(x)$ for arguments between $x_0, x_1, \ldots, x_n$ at which the values $y_0, y_1, \ldots, y_n$ are known.

irrational number: A number which is not rational is called an irrational number. Example: $\sqrt{2}, \pi, e, \ldots$

Klein-Nishina formula: The differential cross section for the scattering of a photon by a free electron (called Compton scattering or the Compton effect) is expressed by the Klein-Nishina formula.

Kortweg-de Vries equation: The non-linear partial differential equation: $u_t + u.u_x + \delta^2 u_{xxx} = 0$ where $u_t = \partial u/\partial t$, $u_x = \partial u/\partial x$ and $u_{xxx} = \partial^3 u/\partial x^3$ is called the Kortweg-de Vries (KdV) equation. A smooth initial wave turns into a series of several profound pulses, each of which is a solitary wave solution of the KdV equation. These pulses proceed freely, collide mutually, interact nonlinearly and then recover their profiles. Zabusky and Kruskal discovered these solutions, in 1965, and named these stable pulses as *solitons.*

London Mathematical Society: The LMS was founded in 1865 for the promotion and extension of mathematical knowledge and is now the major British society for mathematicians. Its principal activities are the publication of periodicals and books and the organization of meetings and conferences. The *Proceedings*, the *Journal* and the *Bulletin* of the LMS are its periodicals. The LMS also publishes the translations of the *Russian Mathematical Surveys*, *Izvestia: Mathematics* and *Sbornik: Mathematics* in collaboration with the Russian Academy of Sciences and Turpion; and in collaboration with the American Mathematical Society publishes the translation of the *Transactions of the Moscow Mathematical Society.* The LMS publishes a monthly *Newsletter*, except in August. The *LMS monographs* are published by the Oxford University Press. The *LMS Lecture Notes Series* and the *LMS Student Texts* are published by the Cambridge University Press. The *History of Mathematics Series* is published by LMS jointly with the AMS. The book *Ramanujan Letters and Commentary* by Bruce C. Berndt and Robert A. Rankin [XI] belongs to this series. The present President of the LMS is Prof. J.M. Ball, F.R.S. (Details may be obtained from the Society's Web site: http://www.lms.ac.uk).

magic square: A magic square is a square array of (usually distinct)natural numbers in which the sum of the numbers in each row, column or diagonals (both the diagonal and the skew-diagonal) are equal. In some instances, the requirement on the two diagonal sums is dropped.

Master of Trinity College: The present Master of Trinity College, Cambridge, is Prof. A.K. Sen, FBA, the economist.

mathematical induction: It is a method of providing proofs to mathematical results based on the principle of mathematical induction which is described as follows: An assertion $A(n)$, depending on a natural number n, is regarded as proved if $A(1)$ has been proved and if for any natural number n the assumption that $A(n)$ is true implies that $A(n+1)$ is also true. This is usually called the first principle of induction. The second principle of induction is described as follows: An assertion $A(n)$ is regarded as proved if $A(1)$ has been proved and if the assumption that $A(k)$, for $1 \leq k < n$, is true implies that $A(n)$ is true.

natural numbers: $1, 2, 3, 4, \ldots$ These are also called positive *integers*.

negative numbers: The equation $x + 1 = 0$ has a negative root, namely -1. (Due to the prevalent opposition to the extension of the notion of numbers, some preferred to say, in the 17th century, that the equation had no root at all!) Thus, the example given shows that the additive inverse of any positive number is a negative number.

neighbourhood: of a point z is an *open set* containing that point.

neutron star: A star that has undergone gravitational collapse where electrons and protons compress to form a neutron core.

Nobel Laureate: A winner of the Nobel Prize is called a Nobel Laureate. The Prize was instituted by Alfred Nobel, the Swedish Chemist, who discovered dynamite. Every year Nobel Prizes are awarded for outstanding work in Physics, Chemistry, Physiology or Medicine, Literature and Peace. However the Prize is not awarded for Mathematics.

nucleus: The positively charged core of an atom, consisting of protons and neutrons. The proton carries one positive electric charge while the neutron has no electric charge.

one dimensional heat equation: The equation: $a^2 u_{xx} = u_t$, where $u_{xx} = \partial^2 u/\partial x^2$ and $u_t = \partial u/\partial t$ is called the one-dimensional heat equation. It is a partial differential equation.

ordinary differential equation: An equation involving one dependent variable and its derivatives with respect to one or more independent variables is called a differential equation. An ordinary differential equation (ode) is one in which there is only one independent variable so that all the derivatives occurring in it are ordinary derivatives. Examples of odes are: $\frac{dy}{dt} = -ky$; $m\frac{d^2y}{dx^2} = -ky$; $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - p^2)y = 0$ where the dependent variable in each of these equations is y, the independent variable is either x or t; and k, m and p represent constants. (see partial differential equation).

partial differential equation: A pde is one involving partial derivatives of one or more dependent variables with respect to one or more of the independent variables.The Laplace equation: $u_{xx} + u_{yy} + u_{zz} = 0$, the heat equation: $a^2(u_{xx} + u_{yy} + u_{zz}) = u_t$ and the wave equation: $a^2(u_{xx} + u_{yy} + u_{zz}) = u_{tt}$, where $u \equiv u(x, y, z, t)$ is a function of the three rectangular coordinates of a point in space and time, are classic examples of partial differential equations. (see ordinary differential equations; one dimensional heat equation).

partition: A partition of a positive number n is a representation of n as the sum of any number of positive integral parts. For example: $5 = 4+1 = 3+2 = 3+1+1 = 2+2+1 = 2+1+1+1 = 1+1+1+1+1$. Thus 5 has 7 partitions. We denote the number of

partitions of n by $p(n)$. Thus $p(1) = 1, p(2) = 2, p(3) = 3, p(4) = 5, p(5) = 7$ etc.
prime number: A *natural number* that is larger than 1 and has no factor other than 1 and itself is called a prime number. The *sequence* of primes are: $2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, \ldots$ The number 2 is the only even prime number. Prime numbers of the form $2^n - 1$ are called *Mersenne numbers*, after the 17th century French Franciscan monk Marin Mersenne (though they were studied earlier by Euclid).
prime number theorem: For every $x > 0$, let $\pi(x)$ denote the prime counting function giving the number of primes that are less than or equal to x. Then $\pi(x)$ is asymptotically (that is, as $x \to \infty$) equal to $\frac{x}{\log x}$. Carl Friedrich Gauss was perhaps the first mathematician who had this result as an unproved hypothesis and more than a hundred years later the Belgian Charles de la Vallée - Poussin and the Frenchman Jacques Salomon Hadamard proved independently that

$$\pi(x) = \int_2^x \frac{dt}{\log t} + O\left(xe^{-A\sqrt{\log x}}\right).$$

Here, $O(\ldots)$ refers to so-called 'big Oh' notation: in general, $O(f(x))$ represents an unspecified function of x which grows not faster than some constant times $f(x)$.
quantum mechanics: Classical mechanics, in general, deals with the mechanics of macroscopic objects. This fails at the microscopic level of the atom and at lower levels. The mechanics at the microscopic level is called quantum mechanics.
Question 330 of Prof. Sanjana: Prof. Sanjana in the Journal of Indian Mathematical Society (IV, 1912, 59 – 61) remarks that it is not easy to evaluate the series:

$$\frac{1}{1^n} + \frac{1}{2}\frac{1}{3^n} + \frac{1.3}{2.4}\frac{1}{5^n} + + \frac{1.3.5}{2.4.6}\frac{1}{7^n} + \cdots \text{ad inf.},$$

if $n > 3$. Ramanujan has summed the series for *all* values of n. In his paper, Ramanujan [III] quotes *Williamson, Integral Calculus* and *Carr's Synopsis, 2295.*
Ramanujan Mathematical Society: Founded in 1985 by a group of Mathematicians in South India, led by Prof. R. Balakrishnan then at the National College, Tiruchi. The *Journal of the Ramanujan Mathematical Society* and a *Newsletter* of the RMS are its regular publications. An annual meeting of the RMS is held in summer every year.
rational number: A number of the form m/n, where m and n are integers and $n \neq 0$ is called a rational number.
relatively prime or coprime numbers: Two numbers m and n are said to be relatively prime if their greatest common divisor is 1. In this case, we also say that m and n are coprime.
Round numbers: A number is described in popular language as *round* if it is the product of a considerable number of complimentary small factors. Thus $1200 = 2^4.3.5^2$ and $2187 = 3^7$ are examples of round numbers – the later more round than the former. The question proposed by Hardy and Ramanujan for themselves is the problem: *How many prime factors may one expect to occur in a random large number n ?*, which they solved. (Ref. [XII], ch. III).
Riemann hypothesis: All nontrivial zeros of $\zeta(s)$, $s = \sigma + it$, are on the line $\frac{1}{2} + it$. This is the famous Riemann hypothesis which has not been proved so far.

Riemann zeta function: The Riemann zeta function is defined as the series:

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s},$$

where s is a complex number with $Re(s) > 1$.

sequence: By a sequence we mean the values of a function f defined on the set $\mathbf{Z}_+$ of all positive integers. If $f(n) = a_n$ for $n \in \mathbf{Z}_+$, we use the notation (a_n) to denote the sequence whose elements are $a_1, a_2, a_3, \ldots$ The elements of the sequence are also called the terms of the sequence. The number a_n is called the nth term of the sequence. Note that the terms need not be distinct.

set: A set is a collection of well defined objects. For example: $\{1, 2, 3\}, \{a, b, c, d\}$, etc.

singularity: If a function of a complex variable $f(z)$ possesses a derivative at some point z_0 and its neighbourhood, in the complex z plane, then the function is said to be *regular* at z_0 and z_0 is said to be a *regular point.* Functions which have at least one regular point are called as *analytic functions.* A point at which a function is not regular is said to be a *singular point*, or merely a *singularity.* For example, the function $f(s) = s^2/(s^2 - 1) = s^2/(s - 1)(s + 1)$ has $s = 1, -1$ as its singular points. The function $f(s)$ is an analytic function which is singular at 1 and -1 and regular at all other points.

solar mass: The Sun is the smallest star that is closest to the Earth. Its mass, 1.985×10^{30} Kilograms, is taken as the unit of mass in astrophysical studies.

solitons: Solitons are essentially elementary excitations in non-linear dissipative systems. (see Kortweg-de Vries equation above).

Statistical Mechanics: This subject was specially devised for the treatment of complicated (many-particle or many-body) systems having many degrees of freedom. The principles of statistical mechanics permit reasonable predictions of the future condition of such a complex system to hold on the average, starting from an incomplete knowledge of its initial state.

stellar evolution: Stars are sources of abundant energy which is produced by nuclear reactions taking place in their interior – the carbon-nitrogen cycle or the hydrogen-helium fusion reaction cycle. The study of what happens to a star as it goes on emitting radiation is the subject matter of stellar evolution. In some cases the star ends up as a white dwarf and in some others as a black hole!

string: The string (in high energy physics) is a one dimensional object, a mathematical curve. Both open strings, which have end points and closed strings, which from a topological view point are circles, are considered. Since a string is free to oscillate it has an infinite number of harmonics, which correspond to an infinite number of particle states.

superstring theory: The fundamental constituents of matter are considered as tiny strings in high energy physics. The theory which treats bosonic (integral spin) and fermionic (half-integral spin) strings as members of the same multiplet (i.e. on an equal footing) is called superstring theory.

symmetric function: A function that remains unchanged under any permutation of its independent variables. For example: $f(x, y) = xy$ is a symmetric function in x and y.

theta function: The four θ- functions of Jacobi are:

$$\theta_1(x,\omega) = i\sum_{n=-\infty}^{\infty}(-1)^n q^{(n-1/2)^2} e^{i\pi(2n-1)x}$$

$$\theta_2(x,\omega) = \sum_{n=-\infty}^{\infty} q^{(n-1/2)^2} e^{i\pi(2n-1)x}$$

$$\theta_3(x,\omega) = \sum_{n=-\infty}^{\infty} q^{n^2} e^{i\pi(2n)x}$$

$$\theta_4(x,\omega) = \sum_{n=-\infty}^{\infty}(-1)^n q^{(n-1/2)^2} e^{i\pi(2n)x}$$

with $q = e^{i\pi\omega}$, Im $\omega > 0$. All the four θ-functions are entire functions of x and are all periodic. The period of θ_1 and θ_2 is 2 and that of θ_3 and θ_4 is 1.

transcendental number: A number which is not algabraic is said to be transcendental. Example: e, π, etc.

trigonometric series: A series of the form

$$\frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx).$$

a_n, b_n are called the coefficients of the series.

unit circle: A circle of unit radius around the origin, in a two-dimensional real or complex plane, is called the unit circle.

Waring's problem: For every $k \geq 2$, there is a number $r \geq 1$ such that every natural number is a sum of at most r kth powers.

White dwarf: A star that has exhausted most of its nuclear fuel and has shrunk to a very small size. This is sometimes the final stage of evolution of a star.

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Srinivasa Ramanujan (1887-1920) hailed as the greatest Mathematician of India and compared to all time greats, Euler, Gauss and Jacobi, for natural genius, is an eternal source of inspiration, especially for the student of mathematics. This book presents a brief biographical account of his life and glimpses into his work on highly composite numbers, elliptic functions, partitions, hypergeometric series, continued fractions, mock theta functions and other topics.

A unique feature of this book is that it provides complete lists of all the papers pertaining to Ramanujan in the Wren Library of Trinity College, Cambridge and the letters and other material about Ramanujan with the National Archives, New Delhi and the Tamil Nadu Archives, Chennai. In the words of Prof. Bruce C. Berndt, who is the world expert on the Notebooks of Ramanujan, this book 'significantly adds to our knowledge about Ramanujan and his work'.

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